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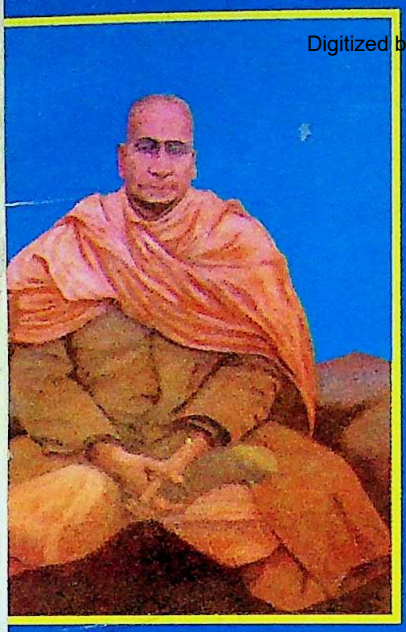

प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका

खण्ड 12, अंक 1, 1998

Journal of Natural & Physical Sciences

Vol. 12, No. 1, 1998

PRE-PROCEEDINGS

No. 1-2

of the

64th Indian Mathematical Society Conference

and

Symposia

on

Traditions of Vedic Mathematics & Applications

and

ed Point Theory in Non-Linear Analysis & Applications

December 19-22, 1998

Department of Mathematics and Statistics

urukula Kangri Vishwavidyalaya, Hardwar 249404

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका
Journal of Natural & Physical Sciences

शोध पत्रिका पटल

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उपाध्यक्ष	एस. एल. सिंह डीन/प्राचार्य
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Printed at : Kiran Printing Press, Kankhal-Hardwar Phone : 0133-415975

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No.F.2-M/98

30th November, 1998

Dear Dr. Dharm Pal,

The President of India, Shri K.R. Narayanan, is happy to know that the Gurukula Kangri Vishwavidyalaya, Hardwar is organising the 64th Annual Conference of the Indian Mathematical Society (IMS) and a Symposia on "Traditions of Vedic Mathematics & Applications" and "Fixed Point Theory in Nonlinear Analysis & Applications" from 19th to 22nd December, 1998.

The President extends his warm greetings to the participants and the organisers and wishes the Conference and Symposia all success.

With regards,

Yours sincerely,

(S.N. Sahu)

Dr. Dharm Pal,
Chairman, Reception Committee &
Vice-Chancellor,
Gurukula Kangri Vishwavidyalaya,
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Gurukula Kangri Vishwavidyalaya, Hardwar

Prof. Sher Singh
Visitor

☎ 011 - 6851718
011 - 6859234
011 - 6857711
Fax : 011 - 6522522

M-14, Saket
New Delhi 110 017

Date 10-12-98

Dear Dr. Dharam Pal:

I am happy to know that a symposia on Traditions of Vedic Mathematics and Applications is being organized during the 64th IMS Conference (19-22 Dec 1998) at Gurukula Kangri Vishwavidyalaya, Hardwar. The IMS is a premier academic body of India dedicated to the cause of Mathematics & Mathematics Education. Therefore its conference is of valuable importance to the society and mankind.

I wish very fruitful deliberations of the conference and symposia on Vedic Mathematics and Fixed Point Theory.

Sincerely,

Professor Sher Singh
Visitor, GKV &
Chairman, International Dayanand Ved Peeth

Dr. Dharam Pal
Chairman, Reception Committee, 64th IMS Conf. &
Vice-Chancellor, Gurukula Kangri Vishwavidyalaya
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OM

Gurukula Kangri Vishwavidyalaya, Hardwar

Surya Dev
Chancellor

Phone : 3264129
1862, Cheerakhana
Maliwara, Delhi 110 006

Date : 12-12-98

Dear Dr. Dharam Pal:

I am happy to know that the 64th Annual Conference of the Indian Mathematical Society is being organized in Gurukula Kangri Vishwavidyalaya during Dec. 19-22, 1998 under your dynamic leadership. Perhaps, it is the first time that the Vishwavidyalaya is organizing such a conference of vital importance in Science College. Further it gives me immense pleasure to know that Professor S. L. Singh, Principal Science College and the Local Secretary of the conference is also organizing a symposium on Vedic Mathematics. Vedic sciences are an integral part of Indian culture and traditions. So the vision behind organizing the symposia and conference in GKV is highly compatible to the vision of the founder of the GKV Svami Shradhanand Ji Maharaj.

I send my greetings and good wishes for a grand success of the conference and symposia.

Sincerely,

Surya Dev
Chancellor, Gurukula Kangri Vishwavidyalaya

Dr. Dharam Pal
Chairman, Reception Committee, 64th IMS Conf. &
Vice-Chancellor, Gurukula Kangri Vishwavidyalaya
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Journal of Natural & Physical Sciences Vol. 12 (1998), No. 1, 1-3

स्वागत – भाषणम्

डॉ0 धर्मपालः

स्वागताध्यक्षः कुलपतिश्च,

गुरुकुल कांगड़ी विश्वविद्यालयः हरिद्वारम्

अमरहुतात्मना स्वामिश्रद्धानन्देन संस्थापिते गुरुकुलकांगड़ीविश्वविद्यालयस्य गणितविभागे समायोज्यमाने गणितविद्वत्सम्मेलने गुरुकुलभुवमागता मान्याः कुलाधिपतयः श्रीमन्तः सूर्यदेवमहोदयाः, मुख्यातिथयः, भारतीय–गणित–परिषदो मान्या अध्यक्षाः प्रो0 डॉ0 बी.के. लहिरी महोदयाः, परिषदः सचिवाः प्रो0 एच.सी. खरे महोदयाः, आचार्यपदे शोभमानाः प्रो0 वेदप्रकाशशास्त्रिणः, सर्वे संकायाध्यक्षा, विभागाध्यक्षा, प्राध्यपकाः, कुलसचिवाः प्रो0 एस.एन. सिंह महोदयाः, भारतीयविश्वविद्यालयेभ्यः शोधसंस्थानेभ्यस्तथा दूरदेशीयविश्वविद्यालयेभ्यः समागताः गणितविद्यापारङ्गता विद्वांसः, सम्मेलनस्य निदेशका विज्ञानमहाविद्यालयस्य प्राचार्याः प्रो. एस. एल. सिंह महोदयाः, सर्वे विद्वांसश्च।

अद्य युगपदेव समागतान् भवतो विलोक्य सर्वेषां गुरुकुलवासिनां मनांसि सानन्दानि सन्ति। गुरुकुले प्रारम्भकालादेव विज्ञानविषयाणां शिक्षाव्यवस्था समुचिता आसीत्। सम्प्रति विश्वविद्यालये अस्मिन् सप्तसंकायाः सन्ति। आधुनिकविज्ञानविषया अत्र वेदानां परिप्रेक्ष्ये पाठ्यन्ते। अत्र गणितविभागे वैदिकगणितस्यैकं प्रश्नपत्रमध्याप्यते। अत्रत्याः प्राध्यापकाः प्राचीनशास्त्रेषु गणितविद्यामन्विष्य शोधगोष्ठीषु स्वशोधपत्राणि पठन्ति।

अस्माकं कृते प्रमोदावहः सुदिवसो यदद्य सर्वेऽपि विद्वांसो भवन्तो गणितविषये विभिन्नान् विचारान् प्रस्तोष्यन्ति विदुषां मनसि गणितविद्यानुरागं च जनयिष्यन्ति।

अङ्कगणितप्रसङ्गे वेदे विद्यमानं गणितबीजं मया यल्लब्धं तदेव किञ्चिद् प्रकाश्यते।

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वेदविद्यारहस्यविदा महिमावता महर्षिणा देवदयानन्देन पूर्वपूर्वतमानामृषीणां सन्देशमुपदेशं च देवगिरा प्रवदता स्पष्टमेवोदीरितम्। यत् समस्तानां सत्यविद्यानां मूलं वेद एव। अर्थात् मानववेद्या या याः सत्यविद्याः सन्ति ताः सर्वा वेदादेव निःसरन्ति। मनुना स्पष्टमेव निर्दिष्टं यत् "भूतं भव्यं भविष्यं च सर्वं वेदात्प्रसिध्यति"। अर्थात् यत् किञ्चिदतीतं यद् वर्तमानं यच्च भविष्यं तत्सर्वं वेदादेव प्रसिध्यति। एतेन ज्ञायते यद् वेदानां सर्वविद्यामूलत्वे नास्ति संशयकणिकोन्मेषावसरः।

अथर्ववेदे गणितस्य विषये सम्प्राप्तये यो मन्त्रस्तत्र त्रिधा गणना कृता। एका संख्या अन्यया संख्यया मिलित्वा का भवति। इत्यस्मिन् प्रश्ने समुद्गते सति एका च दश च मिलित्वा एकादश। यद्येका दश गुणिता भवति तर्हि दश। अस्मिन्नेव मन्त्रे चैकादश संख्याया द्विगुणत्वं त्रिगुणत्वं चतुर्गुणत्वं पञ्चगुणत्वं षड्गुणत्वं सप्तगुणत्वं अष्टगुणत्वं नवगुणत्वं दशगुणत्वं सहस्रगुणत्वं च द्रष्टुं शक्यते।

अत्रमूलरूप एव प्रस्तूयते मन्त्रः –

एका च मे दश च मे।	1+10=11/11X1 = 11
द्वे च मे विंशातिश्च मे।	2+20=22/11X2 = 22
त्रिस्रश्च मे त्रिंशच्च मे।	3+30=33/11X3 = 33
चतस्रश्च मे चत्वारिंशच्च मे।	4+40=44/11X4 = 44
पञ्च च मे पञ्चाशच्च मे।	5+50=55/11X5 = 55
षट् च मे षष्टिश्च मे।	6+60=66/11X6 = 66
सप्त च मे सप्ततिश्च मे।	7+70=77/11X7 = 77
अष्ट च मे अशीतिश्च मे।	8+80=88/11X8 = 88
नव च मे नवतिश्च मे।	9+90=99/11X9 = 99
दश च मे शतं च मे।	10+100=110/11X10 = 110
शतं च मे सहस्रं च मे।	100+1000=1100/11X100 = 1100

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अस्मिन्नेव मन्त्रे निर्दिष्टासु सर्वासु पूर्वपठितासु संख्यासु दशधा गुणीकृतासु तृतीयः क्रमः प्रज्ञातुं शक्यते। यथा

1X10 = 10
2X10 = 20
3X10 = 30
4X10 = 40
5X10 = 50
6X10 = 60
7X10 = 70
8X10 = 80
9X10 = 90
10X10 = 100
100X100 = 10,000

एवमेकस्मिन्नेव मन्त्रे त्रिधात्वं विलोक्य विद्यालये यद् गणितविद्यामूलं वेदे विद्यते। संख्या विषये महीयसी परिगणना संस्कृते विद्यते। प्रथमसंख्यातः समारभ्योत्तरोत्तरं दशधा बुद्धिमुपगता संख्या परार्द्धयं यावत् प्रयाति। यथा

एकं दश शतं चैव सहस्रमयुतं तथा।
लक्षं च नियुतं चैव कोटिरर्बुदमेव च।।
वृन्द खर्वो निरवर्वश्च शङ्खः पद्मं च सागरः।

सूर्यसिद्धान्त विषये अनया संख्यया वर्षादिपरिगणना क्रियते। वेदेषु वेदसम्मतेषु च ग्रन्थेषु गणितविद्या समन्वेष्टव्या गणितविद्यावतां धुरिकीर्तनीयैर्विद्वद्भिः। प्रो0 एस. एल. सिंह महोदयैर्निगदितं यत् सम्मेलनेऽस्मिन् बहूनि शोधपत्राणि वेदसम्मतानि सन्ति इदं वृत्तं भारतीयानां सुखकरं विद्यते।

अन्ते चाहं सर्वेषां मान्यानां वदान्यानां स्वागतम् अभिनन्दनं च व्याहरन् अवाच्यमानन्दमनुभवामि। समेषां सुखबृद्धिं यशोबृद्धिं वयोबृद्धिं वित्तबृद्धिं च भूयो भूयः कामये।

नमस्ते भगवन्नस्तु (सामवेदः)

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Journal of Natural & Physical Sciences Vol. 12 (1998), No. 1, 4-6

FROM THE LOCAL SECRETARY — *S. L. Singh*

This is an overwhelming occasion for our Gurukula family that we are organizing such a big conference for the first time in the history of our Science College, that too on a short time of $3\frac{1}{2}$ months, besides our heavy limitations. The deliberations will encompass frontier areas of mathematics and Vedic mathematics which deals with the golden tradition of Ancient Indian Mathematics. Additional happiness stems from the fact that this conference is very much in tune to the mission of this Vishwavidyalaya as visioned by the founder **Svami Shradhanand Ji Maharaj** (1856-1926). We are highly indebted to the IMS President Professor **B. K. Lahiri**, the IMS council members and especially Professor **H. C. Khare** for giving an opportunity to our Vishwavidyalaya to organize this 64th IMS conference, and further consenting to organize two symposia on Vedic Mathematics & Fixed Point Theory.

Now, our delegates are also aware of our difficulties and limitations. Whatever we have been able to arrange is far less than what our guests and delegates deserve. They are in a city famous for *ashrams* and saints, and we request them to have a saintly way of life for a few days. Mathematicians are known for their dedication to their subject and minimal requirements. They are best known to improve known theories by reducing the requirements, any way. The organizing committee extends a very warm welcome to our distinguished guests, the IMS President, past Presidents, council members and all delegates from India and abroad. The success of our organization depends on your kindness and generosity to embrace upon difficulties that they might have to face here. Another dimension of the success is undoubtedly their active participation is the deliberations of the conference and symposia.

As for as I remember, it was the second week of August 1998, when Professor **H. C. Khare** asked me to organize the present IMS

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conference. Besides the Vishwavidyalaya Calendar and my own programmes incompatible to undertake this penrose work, I started hurried consultations with my departmental colleagues and officers of the Vishwavidyalaya. We were finding only two days (Dec. 21 & 22) for the conference. However, our Vice-Chancellor **Dr. Dharmpal Ji** was enthusiastically in favour of this conference, and, now, I know that he had to sacrifice some of his already fixed schedules too (such as Vice-Chancellors' conference at Indore) and schedule several others. I have been encroaching upon his valuable time during his tight schedules, and it is difficult to find suitable words to appreciate his interest & enthusiasm and express my indebtedness to him for taking enough interest, extending all possible cooperation and maintaining his meaningfull blessings.

Hardwar is a small city and our Vishwavidyalaya is still smaller but both are blessed with good saints and generous persons. The first outside assurance of financial assistance came from Professor **Sher Singh**, Chairman, International Dayanand Ved Peeth, New Delhi and, now, Vishwavidyalaya's Visitor for organizing the conference and symposia on Vedic Mathematics. Although a true Gandhian Prof. Singh has held high positions like central ministership, membership of Planning Commission and currently he is chairman, All India Prohibition Council but this a pleasant coincidence that he is basically a mathematician.

The announcement of the conference could be made in late September' 98 only after the assurance of providing accommodation in Shri Gujrat Dharamshala by Shri Kaku Bhai on a strong recommendation by Shri Ramesh Bhai Thakur of Aryaniwas (Hardwar).

I do not intend to express travails of organizing the conference experienced by some of my departmental colleagues and me during the last 3 months, but I am very happy to express my heartfelt indebtedness to my colleagues, friends, saints and saintly persons from Vishwavidyalaya and outside Vishwavidyalaya, especially from Hardwar & Rishikesh. With the risk of missing many names (from off-campus), I respectfully acknowledge assistance, cooperation and blessings from (in chronological

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order of contacting them) : Dr. **Raj Kumar Singh Rawat**, General Manager, Gurukula Kangri Pharmacy; Dr. **Pranav Pandya**, Director, Brahmavarchas Research Institute, Shantikunj; Anant Shri Vibhusit **Rishikeshvanand Ji Maharaj**, Nirdhan Niketan; Shri **Brahma Swarup Brahmachari**, Sanchalak, Shri Jairam Ashram; Svami **Chidanand Ji Muni**, President, Paramarth Niketan (Rishikesh); Svami **Satyamitranand Ji Maharaj**, Ex-Shankaracharya & Founder Bharat Mata Mandir; Professor **R. Anand**, Managing Director, Altek Devices (Delhi); Shri **Motilal Punjwani**, Managing Director, Hotel Midtown. Further valuable cooperation and assistance extended by Shri **I. D. Joshi** (Principal) and Shri **D. K. Varshney** (Math. Lecturer) of Rishikesh; Dr. **Satish Kumari** (Principal) of Nirdhan Niketan; Shri **N. P. Singh** (City Magistrate, Hardwar); Shri **I. D. Shastri** (Bharat Sadan); Shri **N. D. Malhotra**, Manager, Mandir Mata Vaishno Devi, Hardwar; Shri **Girish Chandra Punetha**, Manager, Pawan Dham, Hardwar; Shri **Chandra Kiran Saini**, Kiran Printing Press, Hardwar; Shri Sunil Gupta, BHEL-Hardwar; **Meera Computers** (Delhi); **Kumar Woods** (Jwalapur) and several others are worth mentioning.

The Organizing Committee thanks the CSIR, New Delhi & International Dayanand Ved Peeth, New Delhi for their financial assistance.

Members of the Organizing Committee have played their roles with full dedication and best of their capability.

Teachers and nonteaching staff, whom we approached, extended their full cooperation and help in this venture. The registery and finance department of our Vishwavidyalaya deserves appreciation for not applying much brakes.

Once again I extend very warm greetings to distinguished guests delegetes and others associated with this conference.

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MATHEMATICS EDUCATION*

B. K. Lahiri**

President, Indian Mathematical Society

Distinguished mathematicians, revered members of the Indian Mathematical Society, delegates, ladies and gentlemen.

It is my great honour and distinction to get this opportunity to address a gathering of mathematical luminaries of our country and abroad. I express my hearty gratitude to the office-bearers of the Indian Mathematical Society for electing me to the prestigious position of the office of the President of the premier Mathematical Society like Indian Mathematical Society. I am extremely honoured by their decision and shall endeavour to keep the upward trend of effective academic activities set up by the past office-bearers and members of the Indian Mathematical Society.

I had the privilege of attending for the first time the annual conference of the Indian Mathematical Society in 1955 held in Cuttack. As far as I remember, at that time the academic activities of the Society throughout the year were confined only to the presentation of papers and some special lectures delivered in the annual conference and there were no other activities in between the annual conferences. But during the last several years, apart from the usual activities in the annual conference, the Society implemented several academic programmes such as special lectures, Memorial lectures etc. throughout the year. Moreover, IMS prizes, AMU prize etc. have been instituted and these are awarded for the best research papers in specified areas presented in the conference. Several prizes after the names of great mathematicians like P.L. Bhatnagar, V.M. Shah etc. are also awarded. Moreover, one of the objects of the Society is to inspire and enthuse young researchers of the country so that they may take the responsibility to maintain the present standard of research outputs, which is at par with the international standard. The credit for all these activities go to the successive office-bearers of the Society who meticulously planned and implemented these activities to enliven the Society throughout the year.

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*PRESIDENTIAL ADDRESS (GENERAL), 64th Conference of the Indian Mathematical Society (19-22 Dec. 1998) Gurukula Kangri Vishwavidyalaya, Hardwar

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It is customary for the President to make some enlightened remark in the conference on the present day mathematics education and research in the country, pointing out defects, if any with possible remedies. However, this topic has been widely discussed during the last two decades by experts and more experienced persons, some of them are in the audience. Therefore, I am afraid to venture on the topic in the presence of so many specialist whose consistent effort explored many sides and who made suggestions for the positive development of mathematics education and research in our country.

Since the President is to some extent duty bound, I attempt to make remark in short only on post graduate education and research.

After graduation, a considerable number of students enquire employment and most of them having failed to secure proper employment turned their attention for admission to post graduate courses. Even after admission, they keep always an eye for securing suitable jobs and as such, for them admission become a stop-gap arrangement. Consequently, there is a dearth of really good, sincere and devotee students in the fields of higher education and the present socio-economic situations in the country is a major factor for this trend.

Due to the large number of intending candidates, holding of admission test in every institutions may be difficult, screening examinations may be held periodically and then perhaps a considerable portion of undesirable infiltration to the higher studies may be curbed.

Presently, the inclination among the students for research works has been increased considerably without perhaps an increase in number of real admirers for research works. This is supported by the fact that many Ph.D's, after obtaining the degree refrained from producing any research paper for a considerable period. This is happening perhaps due to the following causes :

(i) requirement of research papers or Ph.D. degree for undergraduate college teachers (formulated by the University Grants Commission).

(ii) such Ph.D.'s think that their goal to obtain the degree is achieved.

(iii) such Ph.D.'s are unable to adapt themselves with the training they

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receive while working for the Ph.D. degree so that the capacity of carrying on researches independently is not grown.

My experience tells that most of the Ph.D. thesis contain little basic and original results. These contain only generalizations of known results having rare indication of significant and fruitful applications of generalized results. I believe that in a Ph.D. thesis a considerable portion which are original in its true sense should be included.

There is no dearth of talented students in the country and so purely original work is very much possible if the persons concerned are afforded with proper opportunity and facilities. But nowadays due to continuous escalation of prices of research journals and mathematical reviews (MR), even the libraries of the affluent institutions are unable to purchase the same. In the case of reprints, Indian National Scientific Documentation Centre (INSDOC) is providing a marvellous service to the users. But even then they sometimes require more than two months time to provide with the reprints and by this there comes the possibility to loose the trace of the main research problem due to the time gap.

There are several mathematical societies in India almost spread over the country and I believe, any worker is a member of one or more of these societies. We urge upon the Department of Atomic Energy, Government of India through the National Board of Higher Mathematics (NBHM) to sanction proper grants to these societies (the number may not exceed ten), except those who are already purchasing MR, to enable them to subscribe MR annually so that the members of these societies, together they cover most of the research workers of the country, may have regular access to the MR.

Thanking again all of you, I conclude my brief address by wishing a meaningful success of the conference so that the remnant continues for days to come.

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Journal of Natural & Physical Sciences Vol. 12 (1998), No. 1, 10-23

NORMAL STRUCTURE*

B. K. Lahiri**

President, Indian Mathematical Society

The definition of normal structure was introduced by Brödskii and Milman, primarily to solve some geometrical problems. In fact, at a later stage a constant has been defined to be the supremum taken with respect to the convex bounded subsets C of a Banach space X, of the ratio between the Chebyshev radius of C relative to C, and the diameter of C, which has close connection with normal structure. However, besides serious subsequent theoretical investigations on this concept of Brödskii and Milman and related ideas developed later on, the concept of normal structure becomes dominant in the theory of fixed points of operators, specially for nonexpansive mappings. If $T : X \to X$ is a mapping where X is a Banach space then T is called nonexpansive if $\| Tx-Ty \| \leq \| x-y \|$ for all $x,y \in X$. A closed convex subset C of a Banach space X is said to have the *fixed point property* if every nonexpansive $T : C \to C$ has a fixed point.

Here we provide only a glimpse of the idea of normal structure, the presentation, however, could not be exhaustive because of the enormity of the investigations made in this topic recently. We have not touched upon the topics on variants of normal structure, normal structure in spaces besides Banach spaces, sum property, quotient spaces relating to normal structure etc. Unless otherwise stated, X stands for a Banach space.

DEFINITIONS AND EXAMPLES

The following definition was introduced by Brödskii and Milman [5].

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Definition 1 : A convex set $K \subset X$ is said to have *normal structure* if for each bounded closed convex subset H of K which contains more than one point, there is some point $x \in H$ such that

$$\sup \{\| y-x \| \ : \ y \in H \} < \text{diam}\ (H).$$

An equivalent definition has been given by themselves as follows :

Difinition 2 : X has normal structure if and only if there exists in X no bounded non-constant sequence $\{x_n\}$ such that

$$d(x_n, \text{co}\{x_j\}_{j=1}^{n=1})$$

i.e. the distance from the nth element of the sequence to the convex hull of the preceding elements, approaches the diameter of the sequence as $n \to \infty$.

The chebvshev radius of a closed convex subset A of X, denoted by $R(A)$, is defined to be

$$\inf \{\sup \ (\|y-x\| \ : \ y \in A), x \in A\}.$$

It is clear that X has normal structure if $R\ (A) < \text{diam}\ (A)$ [22] for every non-singleton closed bounded convex subset A of X.

Definition 3 : Let K denote the unit sphere in X and let $\delta\ (t)$ be defined in [0, 2] as follows :

$$2\delta(t) = \inf \{2 - \| x+y \| \ : x, y \in K, \| x-y \| \geq t\}.$$

The function $\delta\ (t)$ is known [19, 7] to be *modulus of convexity* and X is uniformly convex if $\delta\ (t) > 0$ for $t \in (0,2]$ {[9, 19] }. The *coefficient of convexity* [12] of X, $\varepsilon_0 = \varepsilon_0(X)$ is

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$$\varepsilon_0(X) = \sup \{t \in [0, 2] : \delta(t) = 0\}.$$

Example 1 [6] : For $1 < p < \infty$, let $x = \{x_n\} \in 1_p$. Let x^+ and x^- be sequences defined as

$$x^+ = \{x_n^+\} \text{ where } x_n^+ = \max\{x_n, 0\}$$

and $$x^- = (-x)^+.$$

Clearly x^+ and $x^- \in 1_p$. If $\|.\|$ denotes the norm in l_p and $1 \le q < \infty$, let $1_{p,q}$ denote the set 1_p with the norm

$$|x| = (\|x^+\|^q + \|x^-\|^q)^{1/q}$$

and let $1_{p,\infty}$ be the set 1_p with the norm

$$|x| = \sup\{\|x^+\|, \|x^-\|\}.$$

Then $|.|$ is a norm for 1_p which is equivalent to the 1_p norm. $1_{p,\infty}$ lacks normal structure, $\varepsilon_0(1_{p,\infty}) = 1$, while $1_{p,1}$ has normal structure and

$$2^{1/p} \le \varepsilon_0(1_{p,1}) < 2.$$

Example 2 [18] . Non-void compact convex sets have normal structure. Uniformly convex Banach space has normal structure. The spaces C [0,1], L^1, L^∞ lack normal structure.

Example 3 [24] . In every Banach space an equivalent norm can be introduced so as to have normal structure.

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We observe a lack of connection between normal structure and reflexivity of the space. In [15] the authors gave an example of a reflexive Banach space lacking normal structure. So reflexivity does not imply normal structure. On the other hand, Zizler [24] proved that each separable Banach space X has an equivalent norm with respect to which X has normal structure. This implies that normal structure does not imply reflexivity.

NORMAL STRUCTURE COEFFICIENTS

Characterization of a Banach space X to have normal structure has been determined in terms of coefficient of convexity and in fact it is known that if the coefficient of convexity is less than one then X has normal structure [12]. The converse is not true as is evident from the space $1_{p,1}$ (Ex 1). Also there is an example of a class of spaces which lack normal structure and have coefficient of convexity one [6] (for example, the space $1_{p,\infty}$, Ex 1). Another important direction is to determine the normal structure coefficients for Banach spaces and in terms of these to determine if a Banach space has normal structure. We proceed to define the coefficients.

Definition 4 [7]: $N(X) = \inf D(C) / R(C)$

where the infimum is taken over all bounded closed convex subsets C of X with more than one member.

Here $D(C)$ denotes the diameter of C and $R(C)$, the Chebyšev radius of C.

Definition 5 [7]. Let $\{x_n\}$ be a bounded sequence in X. The expression

$$\lim_{n} [\sup \{ \| x_m - x_k \| : m \geq n, k \geq n \}]$$

is called the *asymptotic diameter* of the sequence $\{x_n\}$.

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Definition 6 [7] : The *bounded sequence coefficient* of X, denoted by $BS(X)$, is the supremum of the set of all numbers M such that for each bounded sequence $\{x_n\}$ with asymptotic diameter A, there is some y in the closed convex hull of the sequence such that

$$M. \lim_{n} \sup \| x_n - y \| \leq A.$$

Definition 7 [7] : *The weakly convergent sequence coefficient* of X, denoted by $WCS(X)$ is defined as in Definition 6 replacing "bounded" by "weakly convergent"..

These three coefficients are connected by the following relation [7]

$$1 \leq N(X) \leq BS(X) \leq WCS(X) \leq 2 \qquad \ldots(1)$$

provided X is reflexive.

The coefficients $N(X)$, $BS(X)$ and $WCS(X)$ are sometimes called Banach space coefficients or normal structure coefficients.

For certain spaces it has been possible to calculate the above coefficients. For example, if $1 < p < \infty$ then $WCS(l_p) = 2^{1/p}$ [7] and $N(l_2) = 2$ [10]. So we observe that for the space l_2, all the three constants coincide. Also, if any one of these coefficients is greater than one, then X has normal structure [7]. However, there are spaces where all the constants are one although the space has normal structure. For example, let X be the l_2- direct sum of the sequence of spaces $\{l_n\}$, $n \geq 2$, i.e. let X be the collection of all sequences $x = \{x_n\}$, $n \geq 2$ such that x_n is in l_n for each $n \geq 2$ and

$$|x|^2 = \sum_{n} \| x_n \|^2 < \infty.$$

This space has normal structure but all the three constants for this space is one

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It has been proved in [7] that if X is reflexive with modulus of convexity δ, then

$$N(X) \geq (1-\delta(1))^{-1}.$$

So if $\delta(1) > 0$ then $N(X) > 1$ and X has normal structure.

ASYMPTOTIC NORMAL STRUCTURE

The concept of asymptotic normal structure was introduced by Schöneberg and coworkers [4, 21] in connection with the theorem on the existence of fixed points of a nonexpansive selfmap of closed convex bounded subsets of a Banach space.

Definition 8. A Banach space X has *asymptotic normal structure* if for each closed bounded convex subset C of X with more than one member and each sequence $\{x_n\}$ in C for which $\| x_n - x_{n+1} \| \to 0$ as $n \to \infty$, there is a point x in C such that

$$\liminf_n \| x_n - x \| < \text{diam}\,(C).$$

Clearly normal structure implies asymptotic normal structure, but the converse is not true.

For $\beta > 1$, let E_β be the space l_2 renormed as

$$|x| = \max\{ \| x \|_2, \beta \| x \|_\infty \}$$

where $\| x \|_\infty = \sup_n \{ | x_n | \} < \infty$ is the l_∞ norm and $\| x \|_2 = (\sum_{n=1}^{\infty} |x_n|^2)^{\frac{1}{2}}$ is the l_2-norm. Baillon and Schöneberg [4] showed that E_β has asymptotic normal structure if and only if $\beta < 2$ and has normal structure if and only if $\beta < \sqrt{2}$. Thus if $\sqrt{2} \leq \beta < 2$ then E_β has asymtomatic normal structure without having normal structure.

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A question was raised if there exists a reflexive Banach space without asymptotic normal structure. We consider the space $l_{p,\infty}$ with the norm $|x|$ considered in section 2. A short computation shows that for all x in l_p $(1 < p < \infty)$

$$|x| \leq \|x\| \leq 2^{1/p}|x| .$$

Since l_p is reflexive, this inequality shows that $l_{p,\infty}$ is reflexive. The space $l_{p,\infty}$ lacks asymptotic normal structure [8].

We proved [16] a general theorem in this direction by showing that in every infinite dimensional separable Hilbert space an equivalent norm can be introduced for which it lacks asymptotic normal structure.

NORMAL STRUCTURE UNDER ISOMETRY

If X and Y are isomorphic Banach spaces then the *Banach - Mazur distance* from X to Y, denoted by $d(X,Y)$ is

$$d(X,Y) = \inf \|U\| \, \|U^{-1}\|$$

where the infimum is taken over all bicontinuous linear operators U from X onto Y. The distance function $d(X,Y)$ is connected with the above three constants in the following way [7] :

$$N(X) \leq d(X,Y)N(Y),$$
$$BS(X) \leq d(X,Y)BS(Y),$$
and
$$WCS(X) \leq d(X,Y)WCS(Y).$$

It is known [17] that if X and Y are linearly isometric then X has normal structure or asymptotic normal structure if and only if Y has so and that

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$$N(X) = N(Y),\ BS(X) = BS(Y),\ WCS(X) = WCS(Y), \qquad ...(2)$$

The equalities (2) are sometimes very helpful to determine the normal structure coefficients for some spaces.

Example 4. Bynum [7] remarked that $N(L_p\,[0,1])$, $1 < p < \infty$ is difficult to determine and no precise value could be obtained. Recently Prus [20] showed that

$$N(L_p\,[0,1]) = WCS\ (L_p\,[0,1]) = \min\{2^{1/p}, 2^{1/q}\},\ 1/p + 1/q = 1,\ p > 1,$$

but Prus' deduction is too involved. As an application of (2), we can immediately obtain the value of $N(L_2\,[0,1])$. Because $N(l_2) = 2^{1/2}$ {see [7] } and $L_2\,[0,1]$ is linearly isometric to l_2, so $N(L_2\,[0,1]) = 2^{1/2}$.

Example 5. Using (1) and the result of Prus, we see that $BS(L_2\,[0,1]) = 2^{1/2}$.

Example 6. We consider the space R^n_∞ $(n \geq 1)$, where R^n_∞ is the space R_n $(n \geq 1)$ with the maximum norm. Let X be the 1_2-direct sum of the space R^n_∞ $(n \geq 1)$, then it is known [7] that

$$N(X) = BS(X) = 1,\ WCS(X) = 2^{1/2}$$

and also $N(l_2) = BS(l_2) = WCS\ (l_2) = 2^{1/2}$ {see [7] }.

So from (2) it follows that X and 1_2 are not linearly isometric, although one of the coefficients, viz. $WCS\ (X)$ is the same for the spaces.

APPLICATIONS OF NORMAL STRUCTURE

A mapping $T: X \to X$ is called non-expansive if $\| Tx - Ty \| \leq \| x - y \|$ for all $x, y \in X$. A nonexpansive mapping may not have a fixed point [2] . Using normal

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structure, Kirk [13] proved the following basic theorem.

Theorem 1. If X is a reflexive Banach space having normal structure and C is a closed convex bounded subset of X and $T : C \to C$ is nonexpansive then T has a fixed point in C.

It was an open question till 1976 if the normal structure requirement in the theorem was essential. In 1976 Karlovitz [13] considered a space X which is l_2-renormed as

$$\| x \| = \max \{ \| x \|_\infty , \| x \|_2 / \sqrt{2} \}.$$

He considered the subset

$$C = \{ x \in X : x = \{x_i\}, x_i \geq 0, \| x \|_2 \leq 1 \}$$

and proved that it has no w for which

$$\sup \{ \| w\text{-}y \| : y \in C \} < \text{diam } (C)$$

and so X does not have normal structure. But he showed that there exists $T : C \to C$ which is nonexpansive and has a fixed point in C.

Achari and Lahiri [1] proved a fixed point theorem for a mapping $T : C \to C$ where C is a nonempty closed convex bounded subset of a reflexive Banach space X having normal structure where T satisfies the conditions

(i) $\| Tx\text{-}Ty \| \leq \max\{ \| x\text{-}y \| , \| x\text{-}Tx \| , \| y\text{-}Ty \| \}$

for all $x,y \in C$;

(ii) $\sup_{y \in F} \| y\text{-}Ty \| \leq \frac{1}{2} \text{diam } (F)$

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for every non-empty closed convex subset F of C, containing more than one element and mapped into itself by T.

Tiwary and Lahiri [23] relaxed condition (ii) above as

$$\text{(ii)} \sup_{y \in F} \| y - Ty \| < \text{diam}\,(F)$$

and proved the same theorem.

Baillon and Schöneberg [4] proved the above fixed point theorem of Kirk replacing normal structure by asymptotic normal structure.

If X and Y be isomorphic Banach spaces and $d(X,Y)$ be the Banach-Mazur distance from X to Y, Bynum [7] proved that if X is uniformly convex and Y be such that $d(X,Y) \leq WCS(X)$ then each nonexpansive self-map of a closed bounded convex subset of Y has a fixed point.

If X is uniformly convex, Prus [20] defined a constant $\alpha\,(X)$ by

$$\alpha\,(X) = \sup \{ \alpha > 1 : \alpha\,((1-\delta\,(2/\alpha))\ \frac{WCS(X)}{2} + 1 - \delta(1/\alpha)) < 1 \}$$

where δ is the modulus of convexity. Since X is uniformly convex, δ is a strictly increasing continuous function with $\delta(2)=1$ and so the definition of $\alpha(x)$ has a sense. Moreover $\alpha(x)>1$.
He improved the above fixed point theorem of Bynum as follows.

Theorem 2. : If X is uniformly convex and

$$d(X,Y) < \alpha\,(X).\ WCS(X)$$

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then Y has the fixed point property.

Prus noticed that the constant $\alpha(X)$ is not large enough and obtained the value of $\alpha(1_2)$ as 1.0138. He defined a function $f(t)$ by

$$f(t) = \left(\frac{1+t^p}{t^p + (1-t)^p}\right)^{1/p}$$

for $t \in [0,1]$ and proved the following theorem.

Theorem 3. : If $d(1_p, Y) < C_p$ where C_p is the maximum value of $f(t)$ attained in $[0,1]$ for some $1 < p < \infty$, then Y has the fixed point property.

By calculation it follows that

$$C_p > f(1) = 2^{1/p} = WCS(1_p).$$

UNIFORMLY NORMAL STRUCTURE

Gillespie and Williams [11] attempted to obtain the fixed point theorem of Kirk (Theorem 1) by withdrawing the condition of reflexivity of the space and imposing a stronger condition on normal structure, called uniformly normal structure. The Banach space X has *uniformly normal structure* if there exists a number h, $0 < h < 1$ with the property that if C is a closed convex bounded subset of X, then there exists x in C such that

$$\sup \{ \| x-y \| : y \in C\} \leq h.\text{diam}\,(C).$$

Uniformly normal structure clearly implies normal structure but the converse is not true. Consider the Banach space C, the space of all convergent sequences of real numbers with

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$$\| x_i \|_C = \sup \{ | x_i | : i \in N \}.$$

Define a linear, one-one and continuous function g from C to l_2 by

$$g(\{ x_i \}) = \{ x_i / 2^i \}.$$

Now renorm C, for $\{x_i\}$ in C as

$$||| \{x_i\} ||| = [\| \{x_i\} \|_C^2 + \| g(\{x_i\}) \|_{l_2}^2]^{\frac{1}{2}} .$$

In [25] it is shown that this space has normal structure while in [11] it is obtained that this space, i.e. $< C, ||| . ||| >$ does not have uniform normal structure.

Bynum [7] showed that a uniformly convex Banach space has uniformly normal structure but the converse is not true. Renorm the space l_2 by

$$\| \{x_i\} \|_1 = \max \{ |x_1|, (\sum_{i=2}^{\infty} |x_i|^2)^{\frac{1}{2}} \} .$$

Then $(l_2, \| . \|_1)$ is not uniformly convex but has uniformly normal structure.

Gillespie and Williams [11] proved that any nonexpansive selp-map of a closed convex bounded subset of a Banach space with a uniformly normal structure has a fixed point and proved the same result for Kannan-type maps. However, the following theorem of Bae [3] showed that the attempt made by them to remove the reflexivity in Theorem *1* becomes abortive.

Theorem 4. : Any Banach space with a uniformly normal structure is reflexive.

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REFERENCES

1. J. Achari and B.K. Lahiri, A fixed point theorem, Riv. mat. Univ. Parma (4), 6, 161-165, 1980.
2. *D.E.* Alspach : A fixed point free non-expansive map, Proc. Amer. Math. Soc., 82, (1981) 423-424.
3. J.S.Bae : Reflexivity of a Banach space with a uniformly normal structure, Proc. Amer. Math. Soc., 90(2), (1984) 269-270.
4. J.P. Baillon and R. Schöneberg : Asymptotic normal structure and fixed points of nonexpansive maps, Proc. Amer. Math. Soc., 81, (1981) 257-264.
5. M.S. Brödskii and D.P. Milman : On the center of a convex set, Dokl. Akad. Nauk. SSSR N. S., 59, (1948) 837-840.
6. W.L. Bynum : A class of spaces lacking normal structure, Compositio Mathematica, 25, (1972) 233-236.
7. W.L. Bynum : Normal structure coefficients for Banach spaces, Pac. J. Math., 86, (1980) 427-436.
8. W.L. Bynum : An example of a reflexive Banach space lacking asymptotic normal structure, preprint.
9. J.A. Clarkson : Uniformly convex spaces, Trans. Amer. Math. Soc. 396-414, 1936.
10. L. Danzer, B. Grunbaum and V. Klee : Helly's theorem and its relatives, Convexity, Proc. Symp. Pure Math., Amer. Math. Soc., 7, (1963) 101-180.
11. A.A. Gillespie and B.B. Williams : Fixed point theorem for non expansive mappings in Banach spaces with uniformly normal structure, Applicable Analysis, 9, (1979) 121-124.
12. K. Goebel : Convexity of balls and fixed point theorems for mappings with non-expansive square, Compositio Math., 22, (1970) 269-274.
13. L.A. Karlovitz : Existence of fixed points of nonexpansive mappings in a space without normal structure, Pac. J. Math., 66, (1976) 153-159.
14. W.A. Kirk : A fixed point theorem for mappings which do not increase distances Amer. Math. Monthly, 72, (1965) 1004-1006.

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15. W.A. Kirk : Fixed point theory for nonexpansive mapping-II, Contemporary Math., 18, (1983) 121-140.

16. Indrajit Lahiri and B.K. Lahiri : Some structure coefficients for Banach spaces, unpublished.

17. B.K. Lahiri and Indrajit Lahiri : Normal structure under isometry, The Math. Student, 64(1-4), (1995) 218-220.

18. T.R. Landes : Permanance properties of normal structure, Pac.J. Math., 110 (1984) 125-143.

19. S.A. Mariadoss and P.M. Soardi : A remark on asymptotic normal structure in Banach spaces, Rendi. Sem. Mat. Universo, Politech Torino, 44, (1986) 393-395.

20. S. Prus : On Bynum's fixed point theorem, Atti Sem. Mat. Fis. Univ. Modena, 38, (1990) 535-545.

21. H. Schöneberg : Asymptotic normal structure and fixed points of non-expansive mappings, to appear.

22. M.A. Smith and B. Turett : Some examples concerning normal structure in Banach spaces, J. Aust. Math. Soc., 48(Ser A), (1990) 224-234.

23. Kalishankar Tiwari and B.K. Lahiri, Generalisation of a fixed point theorem, Nat. Acad. Math., 3, (1985) 43-46.

24. V. Zizler : Some notes on various rotundity and smoothness properties of separable Banach spaces, Comment. Math. Univ. Carolinae, 10, (1969) 195-206.

25. V. Zizler : On some rotundity and smoothness properties of Banach spaces, Dissertationes Math., Rozpiaway Mat, (1971) 1-37.

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Journal of Natural & Physical Sciences Vol. 12 (1998), No. 1, 24-28

V. M. SHAH - MAN AND MATHEMATICIAN*

N. V. Patel and J. R. Patadia**

Our first association with Professor V. M. Shah was that of a student-teacher relation and it is with great humility and delight that we are penning this write-up. Firstly, we were his students during our M.Sc. studies, thenafter his research students and then as his colleagues in the Department of Mathematics at the Maharaja Sayajirao University of Baroda (M.S. University of Baroda).

During our long association with him, we have seen him as a teacher, as a researcher, as an administrator, as a counsellor to his students and colleagues, a guardian lending helping hand, at times even in personal problems of his students and colleagues, as an organizer and even as a social worker thriving to spread scientific temperament, awareness and attitude in the society.

He truly believes that learning and teaching is a lifelong process and hence even long after his retirement, Professor Shah is academically quite active.

He has a long versatile experience of teaching not only to under graduate and post graduate students at the Faculty of Science, but also at the Faculty of Technology and Engineering of the University.

As a supervisor, we have found him, apart from being mathematically very sound, quite open and frank. Working with him is a pleasure; one always feels encouraged, free and tensionless.

Very considerate, liberal and loving as he is, during the moments

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of depression of a research student, he is a best counsellor and a friend. He is innovative, open to new ideas, and ready to accept the challenge.

He was born in a noble family, hailing from a small village "Sarar" in Baroda district of the erstwhile State of Baroda. He had his Primary education in the same village, and his Secondary and University education in Baroda.

With a brilliant academic career at the M.S. University, he joined (July 1952) the Department of Mathematics, at the M.S. University as an assistant lecturer. Prior to his joining the Department, he was a merit scholar and the Fellow of the Faculty of Science of the University.

He worked under the supervision of Professor U. N. Singh for his Ph.D. degree.

Our joint research work has found place in some of the Journals of International repute e.g. the Journal of the American Mathematical Society, the Journal of the London Mathematical Society, the Journal of Mathematical Analysis and Applications (U.S.A.) etc.

As a Head of the Department of Mathematics, Faculty of Science, M.S. University, he lead the department for a long time. He was a very able administrator. Seeing the rapid industrial growth of Baroda, he could foresee the need of introducing certain courses at the graduate and post graduate level so that students get ample opportunities for jobs, and was the first in the whole state of Gujarat to introduce courses like Linear Programming, Operations Research, Numerical Analysis and Computer Programming. This was done as early as in 1975. Computer Programming was also introduced at the post-graduate level. He knew the potential and ability of each of his colleagues and was always very careful to keep the right person for the right work for the optimum output-at times taking academically and otherwise bold decisions.

He was instrumented in establishing a nice Computer Laboratory in the Department for which he rightly commands all the credit.

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He also commands credit for instituting the degree of M. Phil in the Faculty of Science during his tenure as Dean of the Faculty of Science of the University.

Committed as he is to the cause of education at all the levels, when new mathematics was introduced at school level in seventies in Gujarat State, he took initiative in several directions to offer his own contribution for the educational cause and service to the society at large.

He was the Founder Editor (1972) of a monthly "Ganit Darshan" published with a view to bring home the concepts of new mathematics to school teachers.

At the State level, his contribution to the progress and services of the "Gujarat Ganit Mandal" in nurturing and promoting mathematical activities has been quite significant. He also served the Mandal as its Secretary for four years and later as its President (1979).

He was the Director of the University Computer Centre (since 1983 till his retirement in October 1989). Simultaneously, he was also the Dean of the Faculty of Science (since May 1985 till his retirement).

He wrote an excellent introductory book "Computer Darshan" in Gujarati language for the benefit of students and a common man of the society. Written in a lucid and popular Gujarati language for creating Computer awareness, the book has acclaimed wide popularity.

He is a Founder Secretary of a voluntary organization "Gujarati Vignan Parishad" engaged in Science Popularization activities.

He is also an invitee on the Governing Council of "Community Science Centre" in Baroda. The centre is engaged in organizing programmes for pupularization of Science among masses. He actively participates in the programmes such as seminars, Science fairs, celebration of Science day etc. by the centre.

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He is a Fellow of the National Academy of Science. He is also a Founder Fellow of the Gujarat Science Academy.

He was a member of the UGC sponsored Curriculum Development Centre for Mathematics, set up at the Ramanujan Institute of Madras University.

Professor Shah has delivered series of postgraduate level lectures in several universities. He has also delivered series of lectures during several seminars and refresher courses organized at several university departments.

He was the President of the Mathematics Section (1989-90) of the Indian Science Congress Association, and presided over the Section during the Seventy Seventh Annual Conference of ISCA held under the auspices of the University of Cochin, in February 1990.

He was the President of the Indian Mathematical Society (1990-1991) and Presided over the Fifty Sixth Annual Conference of the Society held under the auspices of the South Gujarat University, Surat in December 1990.

Earlier, served the Society as its Treasurer (1986-1990) and successfully undertook the challenging task of setting this office in order. He also served the Society as Editor of its periodical "Mathematics Student" (1992-1996).

Presently, he (since April 1996) is the Editor of the Journal of the Indian Mathematical Society (JIMS), published by the Society.

He, with the cooperation of like minded people, frequently conducts a one-day or a two-days workshop for Primary school teachers and parents with a view to inspire and enrich them with regard to their Science and Mathematics curriculum, as also to develop scientific tmeper in them.

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In fact a "Primary Teachers Forum" is established in Baroda for this purpose, with Professor Shah as its President.

He always entertained a feeling that our children should be moulded for logical reasoning, and for this purpose efforts should be made to develop a scientific kind of thinking habit among them. He strongly believes that such trained children, in return, would not only become better scientists, but also citizens with scientific temper.

With profound confidence in his belief that Mathematics is the only tool available and handy at the primary stage of our education system for achieving this high goal, he has undertaken a task of preparing a series of five books, coined agewise, for the children of age group 8 to 12.

The series is titled "Majedar Ganit" (Enjoyable Mathematics). Among several other things, the text of the series include a word to the parents, mathematical games, puzzles, fun, some historical tips, some touches on mathematics in ancient India including Vedic mathematics, some-poem-like material etc.

On his retirement, his admirers and well wishers created a "Felicitation Fund" by constituting a committee for the purpose. Admirers and well wishers willingly contributed to the fund. Thenafter, the committee distributed the amoung for various purposes e.g. Institution of Professor V.M. Shah gold medal at the M.S. University, prizes for the Post Graduate Mathematics Students of the University, V. M. Shah prize for the best research paper presented (in Analysis) at the Annual Conference of the IMS, an endownment fund to the Gujarat Ganit Mandal; and creation of a public trust : "Professor (Dr.) V. M. Shah Charitable Trust" at Baroda, for carrying out various socially useful activities of academic nature.

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H. C. KHARE AND MATHEMATICS EDUCATION

S. L. Singh*

Professor Harish Chandra Khare was born on October 5, 1927 at Fatehpur (U.P.). After completing his school and college education at the Government High School, Fatehpur and B.N.S.D. College, Kanpur, he joined the University of Allahabad in 1944 and took his B.Sc. and M.Sc. (Mathematics) degrees in 1946 and 1948 respectively. After a short stay as a Lecture in the Ewing Christian College, Allahabad, in 1948-49, he joined the Department of Mathematics, University of Allahabad, in 1949 as Lecturer. The same year he was selected for the Superior Forest Service but declined the appointment and chose to stay in the University. He went to McGill University, Canada, in January 1956 on a fellowship and obtained his Ph. D. degree from this University in 1960. During his stay at McGill University, he served as a Visiting Assistant professor in the Department of Mathematics. He, also started the Computer Centre and lectured on Numerical Analysis and Computer Programming at McGill University. He returned to the University of Allahabad in 1960. He was appointed Reader in 1964 and Professor of Mathematics in 1976. He was the Dean, Faculty of Science, University of Allahabad (1985-87). He retired from the University in June 1998.

He has to his credit 50 research papers published in reputed journals in India and abroad. His work on Position annihilation was the first of its kind. His publications are in the fields of Theoretical Physics, Magnetohydrodynamics, Plasma Dynamics and Fluid Mechanics. He has lectured at various Universities in India and abroad. He was U.G.C. National lecturer, 1985. He continues to guide research for the Doctor's degree at the University of Allahabad.

He was invited to and participated at the first International Conference on Position Annihilation at Wayne State University, Detroit, U.S.A. in 1965. He on invitation participated in the Seminar on Higher Applied Mathematics at the University of Montreal, Montreal, Canada in 1975.

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He is a Fellow of the National Academy of Sciences, India. He has held various offices in the National Academy of Sciences, India. He had been its General Secretary (1981-84) and (1989-1993), Chief-Section A (1975-80 and since 1985), Managing Editor (1981-84). He was vice-president, National Academy of Sciences, India (1985-1986). He was Foreign Secretary, National Academy of Sciences, India (1986-88).

He was the President Mathematics Section, Indian Science Congress Association, January 1985. He was for several years member of the Council of Indian Science Congress Association. He was the President of Indian Mathematical Society 1986-87 and member of its Executive Committee. He is the General Secretary Indian Mathematical Society.

Professor Khare has been keenly interested in the development of education in the country. He was member of U. G. C. Mathematics Panel and also its Convener. Professor Khare was appointed member, University Grants Commission by the Government of India in 1984 and served till 1987. He was associated with the National Commission on Teachers of the Government of India as a member of its working group.

He was appointed by Government of India a member of the Indo-US Sub-Commission on Education and Culture. He had been a member of the Executive Councils of the Banaras Hindu University, Jawaharlal Nehru University and of the University of Allahabad. He was a member of the Establishment Committee of the National Council of Educational Research and Training, New Delhi. He was a member of the first Executive Council of the newly established Pondichery University. He was a member of the Society of the Indian Institute of Advanced Study, Simla. He was a member of the Working Group on Higher Education Eighth Five Year Plan.

He was member of a University Grants Commission delegation to USSR. The delegation visited USSR in September 1986 to finalise bilateral agreements in the field of higher education. He went as U. G. C. nominee to attend an International Symposium on Informatics and the Teaching of Mathematics in developing countries at Monastir, Tunisia, in February 1986. He attended, on behalf of the U. G. C., the South Asian Region Workshop of Commonwealth Association of Science, Technology and Mathematics Educators at Singapore, in April 1986.

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Professor Khare has been associated with many bodies and committees relating to education in general and mathematics in particular at the National level.

He has been connected with a large number of Universities in various capacities. He took an active part in the restructuring of courses in Mathematics at several Universities. During his service at the University of Allahabad he held many offices.

Professor Khare was invited to the Sixth International Congress on Mathematics Education held at Budapest in July/August 1988 to give a talk on Mathematics and other subjects. He attended this Congress and also visited institutions of higher learning and research in Hungary.

He is now the Treasurer, National Academy of Sciences, India since January 1996. He is the Chief Editor (Physical Science). Proceedings National Academy of Science, India.

Professor Khare is at present Emeritus fellow, University Grants Commission. He is the Coordinator in Mathematics for the preparation of undergraduate Mathematics tele Courses.

Professor Khare was awarded N. R. Sen Memorial Lecutre of Calcutta Mathematical Society 1989 Platinum Jubilee Lecture of Indian Science Congress Association 1991, Distinguished Service award by Mathematical association of India April 1989, P.L. Bhatnagar Memorial Lecture Award. Indian Mathematical Society 1991.

He was member of the Executive of the All India Council of Technical Education New Delhi (1994-1997). He was on the governing body of the Indian Institute of Tropical Meteorology Pune.

He has served on the Committees to prepare panel of names for Vice-Chancellor of various Universities in the country. He declined the Vice-chancellorship of H.N. Bahuguna Garhwal University.

Chairman National Workshop in Vedic Mathematics organised by Rashtriya Ved Vidya Pratisthan, held at University of Rajasthan, Jaipur 25-28 March, 1988.

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ASHOK KUMAR AGARWAL

Virendra Arora*

Professor A. K. Agarwal was born on August 18, 1949. He passed B.Sc. (1969) and M. Sc. (Math., 1971) from Agra University. He is a Ph.D. (1981) from I.I.T., Delhi. He has visited Pennsylvania State University, Mc Master University, Arizona State University, Univ. of Illinois, Auburm University, Wisconsin University, George Washington University, University of California/Minnesota/Qubec/ Victoria etc. His main research field is Number theory and he has published about 50 papers in reputed journals.

The following is a brife write-up of his work by himself.

"I did my Ph.D. in 1981 from IIT, Delhi under the supervision of Prof. H. L. Manocha. The title of my thesis was "Generating relations and differential operator representations". I went to USA in 1984 on a National Scholarship for Higher Study Abroad, Ministry of Education & Culture, Government of India. There under the Supervision of Prof. George E. Andrews, I shifted to partition theory and collaborated with Professors George E. Andrews, David M. Bressoud, Gray L. Mullen (all from the Pennsylvania State University, USA) and Prof. M. V. Subbarao of University of Alberta, Canada on various partition problems mostly related to Rogers-Ramanujan type identities. In our work we not only used known combinatorial objects anti-hook differences and lattice paths but also introduced two new kind of combinatorial parameters called partitions with

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"$n+t$ copies of n" and partitions with "$d(a)$ copies of a". While we used partitions with "$n+t$ copies of n" in obtaining numerous Rogers-Ramanujan type identities, our study of partition with "$d(a)$ copies of a" solved a problem of polynomial algebra posed by R.A. Hultquist, G. L. Mullen and H. Niederreiter. Partitions with "n copies of n" also arise in the solution of the Hard-Hexagon Model of R. J. Baxter.

I have also collaborated with Prof. Willard Miller (Univ. of Minnesota, USA) and Prof. H. M. Srivastava (Univ. of Victoria, Canada) on some special functions problems.

I am continuing my work in partition theory. Very recently I found a bijection between the partitions with "n copies of n" on the one hand and plane partitions on the other. This enabled me to obtain Rogers-Ramanujan type identities for plane partitions which were not found before. Thus I was able to fill in this gap in the theory of plane partitions. Very recently I succeeded in obtaining Ramanujan type-congruences for partitions with "n copies of n". Ramanujan type-congruences for perfect partitions I obtained in 1991 jointly with prof. M. V. Subbarao (Canada)."

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GURUKULA KANGRI PHARMACY

R. K. Rawat*

The Gurukula Kangri Pharmacy is preparing ayurvedic medicines of best quality from last one century. Our products created their own place in the hearts of public due to best standard in quality and high ideality in preparation as mentioned in old Ayurved books. Now, we are one of the largest ayurvedic drugs producer in India.

We are preparing about 400 products. We are also developing new products related to various disease. The potency of our medicines is far better and it is continuously increasing by modern techniques to improve quality. The main branches of medicines, which we are preparing are :

1. Powders : About twenty six
2. Bhasm : About twenty
3. Ras & Vati : About hundred twenty three
4. Pak - Avaleh : About twelve
5. Asava - Arishta : About twenty four
6. Medicines & Cosmetic oils : About fourteen
7. Ghrits : About six

And many new products are under consideration for preparation and testing.From above mentioned products the important products, which have leading sell are Bhimseni Surma, Chyavanprash, Payokil, Shilajeet, Gurukula Chai & Gold Preparations like Vrihat Vat Chintamani, Basant Kusumakar Ras & Yogendra Ras.

Since the unit is situated in the foothills of Himalaya so we

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are much capable to arrange fresh medicinal plants in comparison to other ayurvedic drugs producing units. Since we have plenty of land with us, so we are not only purchasing the fresh medicinal plants but also grow the required plants time to time. The identification of purchased medicinal plants is done by the expert ayurvedic doctors, and only after their approval, the plants or medicinal material is used for preparation. This factor maintained the quality of products in one way.

The another way to improve quality is the methods of prepration of drugs. We are somewhat different with other ayurvedic drugs producing units because we are not compromising with the old methods of preparation of drugs given in standard books of Ayurveda. For this purpose, the strict supervision is done by the Ayurvedic doctors.

The washing of crude material, crushing and grinding of materials or products is done by the help of machines to make work more fast and for other preparative methods, we are strict to Ayurvedic book standard.

Our products specially Asavs and Arishtas are prepared in three to four months by self generating alcohol and not by mixing rectified spirit etc. so their potency towards diseases are very high.

Hence, from all points, we produly can say that due to quality, our products are more potent.

The packaging of medicines is done in different sizes of sealed packs. The packaging label has been printed by the weight of medicine, cost, year and batch number. Before purchasing medicine, it should be ensured that whether the seal is properly intact or not.

The drugs are sold through various agencies all over India.

WE EARN FOR CHARITY : The profit earned the pharmacy is used for charity. The profit is donated for the educatin of students at Gurukula Kangri Vidyalaya (Boys), Hardwar and Kanya Gurukul (Girls), Dehradun.

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VEDIC MATHEMATICS AND GKV

Virendra Arora*

The Gurukula Kangri (GKV) was found in 1902 by Svami Shradhananda Ji Maharaj (1856-1926). Svami Ji had a vision of having interaction between Vedic and modern sciences and this vision was of vital importance, especially because the country was in the hangover of European education and science.

The Vishwavidyalaya was having undergraduate department of mathematics in the College of Science and postgraduate department of mathematics in Ved and Arts College.

Principal S. L. Singh joined this Vishwavidyalaya as Professor of Mathematics in April, 1985, and subsequently the two department were merged in July 1985. He was instrumental in initiating Ph.D. programme (Mathematics) in the College of Science. Other science departments initiated Ph.D. programmes immediately thereafter.

Although Prof. Singh's main research field belongs to Operator Theory & Fixed Point Theory, his first love has always been "Lilavati". In GKV, he seems to be the first to start doctoral level research activities in Vedic mathematics and, of course, in the history of mathematics.

The first Ph.D. enrolment of this Vishwavidyalaya in Ancient Indian Mathematics was done under his supervision at the close of 80's.

Indeed, this was a good beginning for research activities in the science college. Now it is a matter of great pleasure that almost every science department is having a few master courses or doctoral level research work in Vedic sciences.

Our Mathematics Department has already organized two national level symposia on Vedic mathematics and Vedic geometry in 1989 and 1995 under the leadership of Prof. Singh.

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Functional Analysis, Topology, Operator/Optimization Theory, problems related with Computer Applications and Vedic mathematics are the main areas of research activities in mathematical sciences.

The following is a brief detail of Ph.D. theses awarded by the Vishwavidyalaya in this area :

Name	Title & Year of Theses		Supervisor
Ramesh Chand	Study of Siddhänta S'iromaṇi	(1990)	S. L. Singh
Vinod Mishra	Vedic geometry & its applications	(1997)	S. L. Singh
N. B. Singh	Indeterminate analysis & its applications	(1997)	S. L. Singh
Vivek Goel	A study of gaṇitatilaka	(1997)	Virendra Arora

True to his love for ongoing research activities in the tradition of ancient Indian Science, Prof. S. L. Singh has again initiated the present symposium on Vedic Mathematics during this conference which is compatible with Gurukula tradition. As evident from *Abstracts of Paper/Talks (Section N)*, it is a matter of great satisfaction that now other dipartments of the GKV have started taking interest in Vedic Mathematics and related areas. I take this opportunity to express categorically that in particular we and the GKV shall remain indebeted to the path shown by Prof. S. L. Singh.

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Journal of Natural & Physical Sciences Vol. 12 (1998), No. 1, 38-42

GURUKULA KANGRI VISHWAVIDYALAYA AND SCIENCE EDUCATION

Vinod Kumar*

In the middle of the eighteenth century, the kind of educational institutions available in India and institutions which were being established by the British Government and Christian Missionaries were not suitable to cater the educational requirements of Indian Society. The knowledge of modern science subjects could not get appropriate place in the old styled *Pathshalas, Madrasas and Maqtabas.* On the other hand, newly established schools were not having any place for Indian culture. Therefore, there was great need for an educational system, which could have appropriate place for the knowledge of modern subjects, and, at the same time, could generate faith among the students towards Indian culture and heritage. Another gigantic task before educationists was to overcome some superstitions like "*Strishudro nadhiyatan*" and to spread education among all the groups of the society.

Under these circumstances, by establishing Gurukulas, where the students from all walks of the society could equally participate, the Arya Samaj started a new era in the field of education. In these Gurukulas proper arrangements were made to study Snskrit Language, Ancient Literature and Vedic Literature along with the modern science subjects. Following the instructions laid down by Swami Dayanand Saraswati Ji , separate institutions were established for boys and girls. Gurukula Kangri Vishwavidyalaya, established by Swami Sharadhanand Ji in 1902 to give real shape to the dreams of his guru Swami Dayanand Saraswati Ji, proved to be a mile stone among such institutions .

The sole aim was to evolve an alternate system of education, to that British percepation based on English medium, to provide higher education of Vedic Literature, Indian Philosophy, Indian Culture and Literature along with modern subjects through establishing close contacts among the students and the teacher under *'Guru–Shishya'* progeny without distinguishing them on the basis of caste and creed. It helps in character building of the students through their moral and physical development.

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Manifold developments have taken place in the Vishwavidyalaya during the last 98 years. At present there exist four Colleges and six Faculties in the Vishwavidyalaya as listed below :

1. Ved and Arts college.
   - Faculty of Oriental studies
   - Faculty of Humanities
   - Faculty of Management studies

1. Science Collage.
   - Faculty of Science
   - Faculty of Life Science
   - Faculty of Technology

1. Kanya Gurukula Mahavidyalaya, Dehradun

2. Kanya Gurukula Mahavidyalaya, Haridwar.

The Science College :

The education of science subjects initially got started in this Vishwavisdyalaya in 1910. There were mainly two sactions *'Gurukula Vidyalaya'* and *'Gurukula Mahavidyalaya'*. In Gurukula Mahavidyalaya appropriate arrangements were made to provide education of modern science subjects like Chemistry, Physics, Botany and Mathematics through Hindi medium too.

At the same time Chemistry and Physics were introduced in the curriculum of Gurukula Vidyalaya and the medium of teaching, even of science subjects, was Hindi. During this period school level Hindi text books for Chemistry and Physics were written by Master Goverdhan. The efforts were also made to write graduate level textbooks of science subjects in Hindi. "Vanaspati shastra" and "Hindi Chemistry" by Prof. M.C. Sinha and 'Vikasvad' by Prof. V.G. Sathe were the first outcome of these efforts.

Although the foundation of the Science College was laid down in 1921 with the beginning of Ved College with the introduction of science subjects as a part of study, yet in real sense , the work on the development of a separate Science College started with the announcement of government grant

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of rupees one lakh by Dr. Rajendra Prasad (the first President of India) during his convocation address in 1950. The construction work of the building was immediately started . The Science College building was inaugurated by the then Prime Minister Pandit Jawaharlal Nehru on Aug.1,1958. Professor F. C. Trehan was appointed as the founder Principal of the college. Thereafter, the development process was kept on by Principal S. C. Tyagi and Principal Vijaya Shankar. All the Vice-Chancellors and principals have made tremendous efforts for the development of this college. The tenures of the present Vice-Chancellor Prof. Dharam Pal and the present Principal Prof. S. L. Singh have been a golden era in the development history of the Science College. The college is expanding its dimensions in all aspects, be it the construction of buildings, introduction of new courses or completion of research projects funded by various government agencies.

M.Sc, (Maths) and M.Sc. (Microbiology) courses were started in 1964 and 1984 respectively. M.Sc. (Physics) and M.Sc.(Chemistry) were intruduced in 1992. The introduction of such courses at PG level was really a great achievement. But the new technologies were yet to come up. It is in this sequence that Master of Computer Application (M.C.A.) was approved by the U.G.C. in 1993 and M.Sc. Environmental Science in 1994. At present the Science College is taking a leap forward in technological developments as well as in higher education in science. Vedic touch is the salient feature of all these courses.

Faculty & Departmental Profile

I. Faculty of Science

1. Department of Mathematics and Statistics

Courses offered : B.Sc & M.Sc.(Mathematics), (Special emphasis on Industrial Mathematics) Ph.D

Research Areas : Functional Analysis, Topology, Vedic Mathematics, History of Mathematics, Queuing Theory, and Computer Communication Networks.

Faculty : S. L. Singh (Prof. & Dean/Principal), V. Arora (Reader) V. K. Sharma (Reader & Head), M. P. Singh (Reader) P. Pradhan (Lecturer).

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2. Department of Chemistry

Courses offered : B.Sc. & M.Sc. (Chemistry)
(Specialization in Commercial Methods of Chemical Analysis)
Ph.D.

Research Areas : Bio-Inorganic Chemistry, Macrocyclic Chemistry, Electrochemistry, Plant-Medicines, Higher Valent Silver Reaction Mechanism, Falklore and Traditional Medicines, Complex Compounds.

Faculty : R. K. Paliwal (Reader), A. K. Indrayan (Reader)
Kaushal Kumar (Reader), R. D. Kaushik (Reader)
Randhir Singh (Reader & Head), Srikrishan (Lecturer)
Professor's post is likely to be fulfilled soon.

3. Department of Physics

Courses Offered : B.Sc., M.Sc.(Physics) (Specialization in Instrumentation, Design & Maintenance), Ph.D.

Research Areas : Amorphous semiconductors and new materials, Atmospheric physics, Electronics

Faculty : H. C. Grover (Reader), Rajender Kumar (Reader)
Parmanand Prakash Pathak (Reader & Head)

II. Faculty of Life Science

1. Department of Botany & Microbiology

Courses Offered : B.Sc., M.Sc.(Microbiology), Ph.D.

Research Areas : Microbiology, Micropropagation, Enhancement of therapeutic spectrum of indigenous drug and drug plants, Bio-Management of wastewater, Mycology, Plant Pathology.

Faculty : D. K. Maheshwari (Prof. & Dean),
P. Kaushik (Reader & Head),
G.P. Gupta (Sr. Lecturer),
Ramesh Chandra Dubey (Reader), Navneet (Lecturer)

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2. Department of Zoology & Environmental Science

Courses Offered : B. Sc., MSc.(Environment Science), Ph.D.

Research Areas : Water pollution, Ecology, Fishery

III. Faculty of Technology

1. Department of Computer Science

Courses Offered : B.Sc., Master of Computer Applications (MCA), Ph.D.

Research Areas : Distributed Computing, Interconnected Networks, Performance Evaluation, Reliability Engineering.

Faculty : V. Kumar (Prof. & Dean), K. J. Bhatia (Sr. Lecturer)

2. Computer Centre :

Computer centre is equipped with state-of- the-art software and hardware facilities. It has a LAN with UNIX, WINDOWS NT and Novell servers with 40 nodes. All kinds of printer (line, DMP, laser, inkjet) & scanners (B/W, Color) are available in the centre. Internet and E-mail facility are also available.

Process of Admission :

Admission for B.Sc. level in various groups is made on the basis of the marks obtained by the candidate in the subjects required for that particular group. For admission to P.G. courses, a national level entrance exam is conducted at various centres of the country and the admission is granted in the individual P.G. courses.

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Journal of Natural & Physical Sciences Vol. 12 (1998), No. 1, 43-123

Abstracts of Papers/Talks

INDIAN MATHEMATICAL SOCIETY

64th Annual Conference

and

Symposia

on

Traditions of Vedic Mathematics & Applications

and

Fixed Point Theory in Nonlinear Analysis & Applications

December 19-22, 1998

Department of Mathematics and Statistics
Science College

GURUKULA KANGRI VISHWAVIDYALAYA
HARDWAR 249 404

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Abstracts are classified into sections according to the following IMS code:

(A) Combinatorics, Graph Theory and Discrete Mathematics

(B) Algebra, Number Theory and Lattice Theory

(C) Real and Complex Analysis (including Special Functions, Summability and Transforms)

(D) Functional Analysis

(E) Differential Equations, Integral Equations and Functional Equations

(F) Geometry

(G) Topology

(H) Measure Theory, Probability Theory and Stochastic Processes, and Information Theory

(I) Numerical Analysis, Approximation Theory and Computer Science

(J) Operations Research

(K) Solid Mechanics, Fluid Mechanics, Geophysics and Relativity

(L) Electromagnetic Theory, Magneto-hydrodynamics, Astronomy and Astrophysics

(M) Bio-Mathematics

(N) History and Teaching of Mathematics

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A:1

STUDY OF THE SOCIO-ECONOMIC PROBLEM USING COMBINED *FCM*

W. B. Vasantha and S. Uma

Department of Mathematics, I.I.T. Chennai 600036

Here we study the combined effect of the fuzzy cognitive maps for a socio-economic problem. By this method we obtain the combined *FCM* which cannot be done using usual logic. We arrive at the conclusion that poverty causes crime by using combined *FCM*.

A:2

SPLIT BONDAGE NUMBERS OF A GRAPH

V. R. Kulli, B. Janakiram and Radha R. Iyer

Department of Mathematics, Gulbarga University, Gulbarga 585106

Let $G = (V, E)$ be a graph. The negative split bondage number $b_{\bar{S}}^{-}(G)$ of G is the minimum cordinality among all sets of edges $X \subset E$ such that $\gamma_S(G-S) < \gamma_S(G)$ and the split bondage number $b_{\bar{S}}^{-}(G)$ of G is the minimum cardinality among all sets of edges $X \subset E$ such that $\gamma_S(G-X) > \gamma_S(G)$, where $\gamma_S(G)$ is the split domination number of G. In this paper, we initiate a study of these two new parameters.

A:3

CONSTRUCTION OF *q*-ANALYTIC FUNCTIONS

N. Murugesan

Department of Mathematics, Sri Ramkrishna Mission Vidyalaya, Coimbatore 641 020

In all the discrete analytic function theories, discrete analytic continuation is the standard process of defining a discrete analytic function from a domain where only the function is known fully. In q-analytic theory (Harman, 1972) such an operator is evolved from the limit points on the positive coordinate axes to continue to the whole discrete geometric space.

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In this attempt, a function can be continued q-analitically from the basic distorted ray, basic semi raidan ray and its adjacent to the whole of discrete geometric space is evolved. In particular the formula to continue from the basic distorted ray to the first Traint is found. Discussing the anomalies and limitations in the existing theory, This continuation process is an improved version and near to perfection.

In discrete function theories, the modern trend is that a discrete analytic function means only a sequence of values of the function. The function is known fully in the whole space through analytic continuation. In this stream an attempt is made here to initiate a study of q-analytic functions.

A:4

FUZZY COGNITIVE MAPS REPRESENTING SOCIO-ECONOMIC MODEL

W. B. Vasantha Kandasamy

Department of Mathematics, I.I.T., Chennai 600 036

Here we use fuzzy cognitive maps to find the hidden pattern of a socio-economic model with population, crime Economic condition poverty and unemployment as 5 concepts. In this model we study the 3 experts openion and obtain the hidden pattern of the model.

The main conclusion drawn from these experts view is when the population is on the increase we get a fixed point for crime which suggests that increase in population results in increase in crime. We obtain the inter relation between these 5 concepts.

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B:1

OCTANIONS AND 8-DIMENSIONAL VECTORS

M. S. Saroa

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The field of complex numbers has been extended to Octanions (8-dimensional) and an identity, that is, the product of sums of eight squares is equal to the sum of eight squares is oftained. Similar result is oftained through the extesion of vectors to the system of eight dimensional vectors.

B:2

CONSTRUCTION OF FINITE GROUPOIDS USING INTEGERS

W. B. Vasantha and A. Govindarajan

Department of Mathematics, I.I.T. Madras and S.R.M. College, Kattankulathur - 600 036

Here we introduce a new class of groupoids using integers 0, 1, 2, ..., n; (n 3) and study their properties. We further define left identity, right identity and identity in this new class of groupoids. A groupoid defined in this class depends on the pair (t, u); (t, u) ∈ {0, 1, ..., n}. Further it is proved a groupoid obtained using the pair (t, u) is different from the pair (u, t).

B:3

ON BOOLEAN ALGEBRAS

V. R. Chandran

Department of Mathematics, Ramanujan Institute, Chennai 600005

Prof. M. Rajagopalan (Memphis State Univ.) (Personal Communication) has asked for the existence of an atomless Boolean Algebra. In this paper, we Construct such an example using Lattiee theory and Ring theory. We also device an interesting cardlary *vig*[1] There exists no atomless Boolean lattice with dual atoms !

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B:4

GROUP MATRICES

W. B. Vasantha and S. R. Kannan

Department of Mathematics, I.I.T., Chennai - 600 036

Here we introduce a new algebraic structure which does not form a ring or a field or a domain called Group matrices. These group matrices are obtained from the class S of subgroups of a group G. We obtain some characterization theorems and results about them.

Let G be a commutative group. Then $M(G)_{nxn}$ is a semiring in which $A+B = E$ if and only if $A = E$ and $B = E$. Further in this case $M(G)_{nxn}$ is a semi-ring with an additive identity. If G is non commutative group then $M(G)_{nxn}$ is not a semi ring. If $M'(G)_{nxn}$ denotes only the class of normal subgroups of the non commutative group G then $M'(G)_{nxn}$ is a semiring.

B:5

A GROUP HOMOMORPHISM WHICH IS NOT A RING HOMOMORPHISM

Rajiv K. Srivastava

Department of Mathematics & Statistics,
Dr. R.M.L. Avadh University, Faizabad 224 001

A ring homomorphism of one ring to another ring is a group homomorphism under their respective group structures. The converse, it is understood, does not hold, in general. Yet, in literature, there is a dearth of an example of a mapping from a ring to another ring which is a group homomorphism between the respective group structures but fails to be a ring homomorphism.

In this paper, we provide such an example. The said mapping possesses many more interesting algebraic, geometric and functional analytic properties which have been studied in this paper.

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B:6

A LINEAR ORDER ON A SET OF MAXIMAL CHAINS OF A SEMILINEAR ORDER

Shabeena Ahmed

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A linearly ordered set or a chain is a partially ordered set in which any two elements are comparable with respect to the partial order. A semilinear order generalises the notion of a linear order and a C-relation is a ternary (3-place) relation defined on the set of maximal chains of a semilinear order. We construct a semilinear order on the plane and consider a dense set of maximal chains with the underlying C-relation. We then impose a linear order on the set of maximal chains and study the Jordan sets of the automorphism groups of the set of maximal chains. The Jordan sets of the maximal chains with the C-relation have already been studied by S.A. Adeleke and P.M. Neumann. In this paper we give a complete description of the Jordan sets of the maximal chains preserving both the C-relation and the linear order.

B:7

A NEW IDENTITY IN LATTICES

W. B. Vasantha Kandasamy

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Here we introduce a new identity in lattices and call those lattices which satisfy this identity to be a N-lattice, we prove every N-lattice is modular and super modular. Further we prove every distribute lattice is a N-lattice and not conversely.

B:8

SEMI-MODULES OVER SEMI-RINGS

W. B. Vasantha and N. Suresh Babu

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Here we define a new notion called semi-modules over semi-rings.

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We mainly prove that if M is a strict S-semi-module then S is a strict semi-ring. However even if S is a strict semi-ring we prove every S-semi-module over S need not be a strict-semi-module.

Further we define quotient semi-module and obtain some interesting results about them. We finally define tensor product of two S-semi-modules and pose it as open problem to carry over to prove results on tensor product of S-semi-modules.

B:9

CONCATENATION OF CODES WITH RANK METRIC

W. B. Vasantha and R. Sujatha

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The algebraic coding theory is one of the branches of communication theory which plays a vital role in the transmission of messages. To have a reliable transmission of messages, we need codes which can both detect and correct errors. Concatenation of codes is an important method to obtain lengthy codes with good error correcting capability. Here we are the first one to introduce concatenated codes with rank metric. These codes not only transmit the information reliably but also help us in detecting and correcting errors.

B:10

HAMILTONIAN GROUP RINGS

W. B. Vasantha Kandasamy and V. Vaithiyanathan

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Here we obtain conditions under which the group rings are Hamiltonian or otherwise. Jin Xiang Xin was the first one to define generalized Hamiltonian Rings. He calls a ring R to be a generalized Hamiltonian ring if every non zero subring of R includes a non zero two sided ideal.

We prove the group ring Z_2G of a group G over Z_2 which contains an element of order two is not Hamiltonian. Thus Z_2S_n is not an Hamiltonian group ring for every S_n for all n. In case of torsion free groups G the question whether the group ring RG is Hamiltonian remains unsolved.

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B:11

SUPER MODULARITY OF SUBLOOPS IN LOOPS

W. B. Vasantha Kandasamy and S. V. Singh

Department of Mathematics, I.I.T., Chennai 600 036

Here we study when does subloops of a loop form a supermodular lattice. We obtain a characterization theorem for the subloops of a finite loop L to be a supermodular lattice.

The collection S of all subloops of a finite loop L is a supermodular lattice if and only if S does not contain three distinct subloops such that one and only one pair is comparable and S does not contain a set of four distinct subloops S_1, S_2, S_3, S_4, such that $S_1 \cup S_2 = S_1 \cup S_3 = S_1 \cup S_4 > S_1$ and $S_1 > S_2 \cap S_3$, $S_2 \cap S_4$ and $S_3 \cap S_4$.

B:12

FORMAL INTEGRATION IN COMPOSITION NEAR- RING

S. Nabachandra Singh

Department of Mathematics, Manipur College, Imphal 795 008

The purpose of this paper is to introduce a formal integration operator for composition near-ring and to study about its recurrance and about its properties on some well-known composition near-rings.

B:13

A NOTE ON UNITIFICATION PROBLEM

B. N. Waphare and S. S. More

Department of Mathematics, University of Pune

S. K. Berberian raised the open problem "Can every weakly Rickart*-ring be embedded in a Rickart *-ring ? With preservation of *RP's*?" N. K. Thakare and B. N. Waphare gave a partial solution a weakly Rickart-ring A can be embedded in a Rickart *- ring provided there exists a ring K

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such that

(i) K is an integral domain with involution,

(ii) A is a *-algebra over K,

(iii) for any $\lambda \in K - \{0\}$ there exists a projection $e_\lambda \in A$, such that $x \in A$, $\lambda_x = 0 \Rightarrow LP(x) \leq e_\lambda$.

In this paper a more general partial solution of the problem is obtained. More precisely in the above partial solution the integral domainness of K is replaced by arbitrary commutative ring with identity.

B:14

ON STONE POSETS AND STRONGLY PSEUDOCOMPLEMENTED POSETS

M. M. Pawar and B. N. Waphare

Department of Mathematics, University of Pune

In this paper some of the results in Halas [2] are strengthened. The concept of strongly pseudocomplemented poset is introduced and obtained its characterization and established its connection with Stone poset.

B:15

A CRITERION FOR DIVISIBILITY OF NUMBERS

K. K. Velukutty

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There are well known and simple divisibility criterion for numbers like 3, 7, 9 and 11. A uniform rule is not available. An attempt is made here to establish a criterion for and to formulate a unified approach to divisibility. The criterion reads as : N is divisible by r if and only if r is a factor of 10^1-t and the sum of the products of the syllables of N of order 1 and the corresponding multipliers of t is divisible by r.

In many cases this practical procedure is very simple and in the rest not at all tough. Various formulae are introduced in a tabular form for divisibility by smaller numbers; cases like those of 37, 58, 77, 91 and 143 are demonstrated to be simple.

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A two man game is formulated between the offender and the defender such that the defender always wins; certain special cases are demonstrated. This note is important in two aspects : as a unified theoritical development for criterion for divisibility of numbers as well as a recreational game on numbers.

B:16

PROOF OF FERMAT'S LAST THEOREM USING ITS ORIGINATOR'S METHOD

V. K. Gurtu

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Andrew wiles (1995) has published 109-page proof of Fermat's Last Theorem (*FLT*) by identifying it with a structural conjecture in Number Theory involving elliptic curves. However, in the present exposition, we have proved *FLT* for a general odd prime by adopting techniques which were used in Euler's proof of *FLT* for Cube. During the process we have introduced only a minor change in terms of identifying one of the two equations obtained from the governing equation of *FLT* in a slightly different way. But, significantly, this minor change has simplified the proof to such an unimaginable extent that it becomes almost one with the tought process of the originator of *FLT*.

B:17

ON RADICALS OF NEAR-RINGS

Manoj Kumar Manoranjan

Department of Mathematics, B.N.M.V. College, Madhepura 852113

Here the study of radicals of finite near-rings is initiated. The result gives a description of hereditary radicals having hereditary semi-simple classes. Since any radical in the class of associative rings has a hereditary semi-simple class. Also in the variety of not necessarily associative rings hereditary semi-simple classes are quite rase. General radical theory of near-rings paid special attention to the hereditariness of semi-simple classes. Also finite near-rings apply the structure theory of near-rings with DCC on right N-sub groups.

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B:18

A REMARK ON FERMAT'S THEOREM

Mangey Singh

R.S.S. Inter College Jhinzhana, Muzaffarnagar

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C:1

AN EXTENSION OF GENERAL CLASS OF BILATERAL GENERATING FUNCTION OF MODIFIED JACOBI POLYNOMIALS

M. A. Hussain and Faizi Aftab

Department of Mathematics, V.K.S.U., Ara, (Bihar)
Department of Mathematics, Oriental College, Patna 800 006

In this paper, we extend the general theorem on bilateral generating function of modified Jacobi polynomials. Some applications have also been deduced.

C:2

DISTRIBUTIONAL ABELIAN THEOREMS FOR A GENERALIZED STIELTJES TRANSFORM

Anil Kumar Mahato and J. M. C. Joshi

Department of Mathematics, Ranchi University
Department of Mathematics, Kumaon University, Nainital

In this paper an extension of the generalized stieltjes transform

$$F(s) = \Gamma(\rho) s^{m\rho-1} \int_0^\infty \frac{f(t)}{(s^m+t^m)^\rho} dt \quad (m,p > 0)$$

to generalized functions is provided. Some Abelian theorems for the distributional generalized Stieltjes transform are proved and application of the Abelian theorem is given.

C:3

ASYMPTOTIC REPRESENTATION OF WEYL FRACTIONAL INTEGRALS

P. K. Banerji, Deepali Sinha and Megumi Saigo

Department of Mathematics, JNV University, Jodhpur 342 005

Object of the present article is to obtain asymptotic representation

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of Weyl fractional integral, based on a method, employed by Kilbas for the Riemann-Liouville fractional integral. We have obtained the asymptotic representation for the case $\mu_k \neq p\ (j + 1)$ and, further, have formulated the asymptotic representation for n- dimensional fractional integral of $|x| \to \infty$ for the case $\mu_k \neq n + 2j$.

C:4

AN ALTERNATIVE PROOF OF SZEGÖ TYPE RELATION FOR THE SET $\{K_n(\alpha, \beta, k; x)\}$

C. B. Kshatriya and P. G. Andhare

Department of Mathematics, H.P.T./R.Y.K. College, Nashik
Department of Mathematics, RBNB College, Shrirampur

Madhekar and Thakare defined and studied biorthogonal polynomials $\{J_n(\alpha,\beta,k;x)\}$ and $K_n(\alpha,\beta,k;x)\}$, suggested by Jacobi polynomials. In present note we give an alternative proof of Szego type relation for $\{K_n(\alpha,\beta,k;x)\}$.

C:5

ON BAILEY'S FORMULA INVOLVING PRODUCT OF GENERALIZED HYPERGEOMETRIC SERIES

Arjun K. Rathi

Department of Mathematics, Dunger College, Bikaner 334 001

The aim of this paper is to obtain single series expressions for

$$F(-;\ \rho;\ x)\quad F(-;\ \rho+ i;\ -x)$$

for i=1 and -1. For i=0, we have the well known result due to Bailey. The results are derived with the help of Generalized Kummer's theorem very recently obtained by Lavoie, Grondin and Rathie.

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C:6

A NOTE ON KUMMER'S SECOND THEOREM

Arjun K. Rathi and Manish K. Sareen
Department of Mathematics, Dunger College,
Bikaner 334 001

The aim of this research note is to derive the well known Kummer's second theorem from a Bailey's formula involving product of generalized hypergeometric series.

C:7

A UNIFYING APPROACH TO A NEW INTEGRAL TRANSFORM

P. K. Banerjee and D. N. Vyas
Department of Mathematics, MLV Textile Institute,
Bhilwara 311001

The object of the present paper is to define a new integral transform which is quite general in character. Special cases in terms of knwon integral transform are also given. Its relation with fractional calculus has been reported and a table of integral transforms or certain elementary functions is also provided.

C:8

LOCALIZATION OF $\left[\tilde{N}, p_n\right]_k$ SUMMABILITY OF FOURIER SERIES

Syed M. Mazhar
Department of Mathematics, Kuwait University, Safat 13060, Kuwait, P.O. Box 5969

The paper deals with the localization problem for absolute summability of Fourier series.

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C:9

STIELTJES TRANSFORM OF A CLASS OF GENERALIZED FUNCTIONS

P. K. Banerjee, Kuldeep S. Gehlot and Megumi Saigo

Department of Mathematics, JNV University Jodhpur 342 001

Department of Applied Mathematics, Fukuoka University, Fukuoka, Japan 814 080

In this paper a generalization of stiltjes transform, involving Gauss hypergeometric function in the kernel, is extended to generalized functions. Two testing function spaces are constructed, properties of testing function space and its duals are studied. Transformable generalized function are defined and analyticity theorem is proved for the generalized integral transform.

C:10

VALUE DISTRIBUTION OF FOURTH PAINLEVE DIFFERENTIAL EQUATION

S. S. Bhoosnurmath and C. M. Hombali

Department of Mathematics, Karnatak University, Dharwad

In this paper we consider fourth painleve type differential equation

$$ff^{(4)} = f^{(3)} f' - \frac{f^{(2)})^2}{2} + 20f^6 + 96f^5 + 90f^4 + 80zf^3 + 60(z^2 - \alpha)f^2 + B$$

Where B is some constant. Applying Nevanlinna theory we study the value distribution properties of transcendental meromorphic solutions of such equations. For the transcendental mermorphic solutions f of the above equation, we make an interesting observation that the second fundamental theorem of Nevanlinna goes into asymptotic equality.

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C:11

ON THE DOMAIN OF NORMALITY IN ITERATION THEORY

P. Bhattacharyya

Department of Mathematics, IIT, Chennai 600 036.

In this paper we prove some new results on the components of the FATOU SET $F(f)$ which is the set of normality (in the sense of Montel) of the sequence of natural iterates of a function of a complex variable z.

In particular we prove that if f is a finite map, then $F(f)$ cannot have more than two components.

C:12

ON CERTAIN q-SERIES IDENTITIES

T. B. Jagtap

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This paper incorporates a q-series indentity along with its interesting particular cases.

C:13

ON CERTAIN q-SERIES AND CONTINUED FRACTIONS

Remy Y. Denis

Department of Mathematics, Gorakhpur University, Gorakhpur 273 009

Continued fractions representation for certain Mock theta functions have been established. The paper concludes with a brief discussion of the importance of this type of representations.

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C:14

SOME POLYNOMIALS ASSOCIATED WITH A GENERALIZED POLYNOMIAL SET $\{P_n^{\alpha,\gamma}(x,r,s,p,k,\lambda)\}$

P. G. Andhera and T. B. Jagtap

Department of Mathematics, R.B.N.B. College, Shrirampur 413709

Patil and Jagtap studied the polynomials $\{P_n^{\alpha,\gamma}(x,r,s,p,k,\lambda)\}$. Here we define and study some polynomials associated with generalized polynomial set.

C:15

AN ESTIMATE OF A PARTIAL SUM OF THE FOURIER SERIES

Prem Chandra and Aalok Verma

School of Studies in Mathematics, Vikram University, Ujjain 456 010

In this paper, an estimate of a partial sum of the Fourier series of $f \in L_{2\pi}$ at a point under more general conditions has been obtained which leads to some known estimates under weaker conditions imposed upon the generating functions of the Fourier series. An application of this estimate also yield an interesting criterion for the convergence of the Fourier series at a point.

C:16

SOME RESULTS ON MATRIX TRNASFORMS OF H-FUNCTION

Vilas D. Raghatate

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In this paper we have established some new results related to Matrix transforms of H-function the H-function is defined by Innayat Hussain in 1987 is more generalised form of Fox's *H-function*. Many author's have established results on matrix trnasforms of Meijer's *G-function* and also Fox's *H-function* and many other functions.

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C:17

ON THE PARTIAL SUMS OF STARLIKE FUNCTIONS

Sukhjit Singh

Department of Mathematics, S.L.I.E.T., Longowal

Let $S_t^{(1/2)}$ denote the usual class of starlike functions of order ½. If $S_n(z,f)$ denote nth partial sum of f, then it is well knwon that Re $[f(z)/S_n(z,f)] > 1/2$, for all $f \in S_t^{(1/2)}$ and for all z in the unit disc $E = \{z / |z| < 1\}$.

In the present paper, among other things, we give two way generalization of this results and prove : If $f \in S_t^{(1/2)}$ then for all t, $0 \le t \le 1$ and for all $n \in N$, we have (i) $Re[tf(z)/S_n(tz, f)] > 1/2$, $z \in e$ and (ii) $Re[tf(z)/\sigma_n(tz, f)] > 1/2$, $z \in E$. Here $\sigma_n(z, f)$ denote the nth cesaro sum of order i, i=1,2,3.

C:18

SOME BILATERAL GENERATING RELATIONS FOR THE FAMILY OF GENERALIZED POLYNOMIAL SYSTEMS

G. N. Dave

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Earlier, Agarwal and Dave have defined a family of generalized polynomial systems $B_n^B\{(x_m),Y\}$, $B_n^C\{(x_m),Y\}$ and $B_n^D\{(x_m),Y\}$ by means of generating relations involving the generalized hypergeometric functions p_q^F and Lauricella's functions F_B, F_C and F_D. We have already discussed their explict forms, generalized Lauricella forms and n^{th} differential formulae. In this paper several bilateral generating relations of these polynomials has established and their relations with classical bilateral generating relations discussed in detail.

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C:19

ON A SEQUENCE OF FOURIER COEFFICIENTS

K. N. Singh

Department of Mathematics, S.G.R. (P.G.) College Dobhi Jaunpur 222 149

In the present paper we prove a theorem on C_1. (N,p,q) summability of the sequence $\{nB_n(x)\}$ under very general condition.

The C_1. (N,p,q) # (N,p,q). C_1 except when $\{p_n\}$ is defined as silverman.

If q_n=1 for all n, the (N,p,q) means reduces to the (N,p_n) means and if p_n=1 for all n, the (N,p,q) means reduces to the (N,q_n) means of the sequence $\{S_n\}$.

C:20

ON THE ILYEFF - SENDOV CONJECTURE

Prasanna Kumar N.

Department of Mathematics, Mangala Gangotri University 574 109

A conjecture due to Sendov, better known as Ilyeff-Sendov conjecture is stated as follows,

CONJECTURE : Let $P(z)$ be a polynomial of degree $n \geq 2$ with all its zeros in the unit disk $|z| \leq 1$. If a is any one of these zeros, then $P'(z)$ has atleast one zero in the disk $|z-a| \leq 1$.

This conjecture has been verified for all polynomials of degree n, $2 \leq n \leq 5$. In this note, I verify the velidity of the conjecture for polynomials of degree n with atmost five distinct zeros. I reached the solution from a different direction and a shortest route which improves all earlier results on this.

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D:1

AN APPLICATION OF SCHAUDER'S FIXED-POINT THEOREM

B. S. Yadav

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Schauder's Fixed-Point Theorem states that every convex compact subspace of a Banach space is a fixed-point space. By using a variant of this theorem, Lomonosov proved a theorem on the existence of non-trvial invariant subspaces of non-zero compact operators. We discuss in this *TALK* the present state of Lomonosov's theorem in the context of recent developments in the invariant subspace theory.

D:2

ON ADJOINTS AND SPECTRA OF SHIFT OPERATORS ON WEIGHTED SEQUENCE SPACES

Bhopinder Singh

Govt. College of Engineering and Technology, Jammu

For $a > 0$ and for $1 \leq p < \infty$, let $l^p(a)$ be the Banach space of all complex sequences $x = \{x_n\}_{n \in N}$ for which

$$\|x\| = (\sum_{n=1}^{\infty} a^n |x_n| p)^{1/p} < \infty.$$

In this paper, the adjoints and the various parts of spectra of the right shift and left shift operators on $l^p(a)$ have been calculated.

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D:3

FIXED POINT THEOREMS FOR BANACH OPERATORS ON PRODUCT SPACES

A. B. Lohani

Department of Mathematics, Kumaon University, Nainital 263 002

In this paper we introduce the Banach operators on finite product of 2-metric spaces and obtain fixed point theorems for such operators.

D:4

ON CONTINUITY AT FIXED POINTS

R. P. Pant

Department of Mathematics, Kumaon University, Nainital 263 002

The aim of this paper is to provide an affirmative answer to the open question on the existence of a contractive condition which is strong enough to generate a fixed point but does not force the map to be continuous at the fixed point. We do this by using the notion of reciprocal continuity and obtain a common fixed point theorem which significantly improves the known fixed point theorems for four mappings.

D:5

COMMON FIXED POINT THEOREMS FOR R-WEAKLY COMMUTING MAPPINGS OF TYPE (A_f)

R. P. Pant and S. Padaliya

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In this paper is to study some consequences of recently introduced notion of R-weak commutativity of type (A_f) by Pathak *et al.* [10]. We obtain some fixed point theorems for compatible and non-compatible mappings. Also we have employed the notion of pointwise R-weak commutativity of type (A_f) is a necessary condition for the existence of a common fixed point of contracive type mappings.

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D:6

ON COMMON FIXED POINTS OF ASYMPTOTICALLY REGULAR MAPPINGS

Praveen Kumar Shrivastava, N. P. S. Bawa, Vivek V. Raich and Pankaj Singh

Govt. P. G. College, Balaghat, Govt. Science College, Rewa

Without considering the usual sequence of successive approximations (an idea developed by Rhoades and Sessa *et.al.*), the results of this paper are established in metric space (X, d) in order to show the existance of common fixed points under following contractive condition;

$$d(Sx, Ty) \leq a_1 d(Sx, Ax) + a_2 d(Tx, Ax) + a_3 d(Sy, Ay)$$

$$+ a_4 d(Ty, Ay) + a_5 d(Sx, Ay) + a_6 d(Tx, Ay)$$

$$+ a_7 d(Sy, Ax) + a_8 d(Ty, Ax) + a_9 d(Ax, Ay)$$

for all x, y in X.

The main result extends and generalizes essentially known results of Das and Naik, Fisher, Jungck, Rhoades and Sessa et. al., Singh and Tiwari and Several others.

D:7

COINCIDENCE POINTS FOR HYBRID CONTRACTION

Praveen Kumar Shrivastava, N. P. S. Bawa and Vivek V. Raich

Govt. P. G. College, Balaghat, Govt. College, Amarpatan, Satna

Using the idea developed by Singh et. al. sufficient conditions for the existance of coincidence points for a very general type of contractive condition involving two pairs of mappings. Where each pair consists of a single-valued and a multi-valued mappings (known as hybrid contractin) are obtained. Main result of the paper extends and generalizes the results of Naidu, Sastry, Naidu and Prasad.

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D:8

ON CHARACTERIZATIONS OF FIXED POINT OF CONTINUOUS MAPPINGS

Renu Chugh and Sanjay Kumar

Department of Mathematics, M.D. University, Rohtak

Some results concerning necessary and sufficient conditions for the existence of fixed points of continuous mappings in metric spaces have been proved by Dien and Liu, Xu and Cho using compatibility. These results generalize, improve and unify the corresponding results of Chang, Fisher, Dien and Jungck. In this paper we obtain the same result but with a generalized contractive condition which gives in particular the result of Liu, Xu and Cho. Our results generalize, improve and unify corresponding results of Chang, Fisher and Jungck.

D:9

ON THE KNASTER - KURATOWSKI - MAZURKIEWICZ - SHAPLEY - FAN THEOREM

D. D. Sharma

Department of Mathematics and statistics, GKV Hardwar 249 404

For almost five decades *KKM theorem* have been riding a wave that has enveloped many areas of mathematics, social and natural sciences in its power, creativity and expanse. Traveling far beyond the traditional bound of mathematics and science to distant shores of popular culture, this wave have captured the attention and enthusiasm of a worldwide audience.

In this *talk* our aim is to present a survey of generalizations of *KKM* theorem and some of its applications in nonlinear analysis.

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D:10

COMPLEX 2-BANACH ALGEBRAS

E. Keshava Reddy, S. Bhattacharya and S. N. Lal
Department of Mathematics, Banaras Hindu University, Varanasi

In this paper we have introduced the notion of complex 2-Banach algebras and among several other result it has been shown that if x is in a Complex 2-Banach algebra then the spectrum $\sigma(x)$ is a closed bounded non-empty subset of the Complex plane C.

D:11

COMMON FIXED POINTS OF SET-VALUED MAPPINGS

P. P. Murthy
Department of Mathematics, Arignar Anna Govt. Arts College, Karikal 609605

The purpose of the present paper is to study continuation of the author of this paper with Singh and Singh ([11]). These theorems in this paper are indeed extension of the Gregus [06], Fisher and Sessa [5], Diviccaro, Fisher and Sessa [02], Murthy, Cho and Fisher and many results of set-valued mappings.

D:12

THE COMETRIC PROJECTION AND FIXED POINTS

Geeta S. Rao
Ramanujan Institute for Advanced Study in Mathematics
University of Madras, Chennai 600 005

Let X be a normed linear space over the real numbers and let G be a nonempty subset of X. Let $x \in X \setminus \bar{G}$ and $g_0 \in G$. Then g_0 is an element of best coappromimation to x if $\|g_0 - g\| \leq \|x - g\|$ for every $g \in G$. If R_G is the map associating with x, the set of all elements of best coapproximation to x, then R_G is called the cometric projection. Some results concerning cometric projection and fixed points are presented. Releated maps are defined and discussed. Certain connected ideas are elaborated.

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D:13

A NONLINEAR ERGODIC THEOREM FOR MULTIVALUED OPERATORS

S. L. Singh

Department of Mathematics and Statistics, GKV Hardwar 249 404

The first nonlinear ergodic theorem, due to J.-B. Baillon, says that sequences of iterates of a nonexpansive operator on a Hilbert space (with some condition) are almost convergent to a fixed point of the operator. The purpose of this talk is to offer a brief report on nonlinear ergodic theory, and to present an ergodic theorem for multivalued operators.

D:14

A FIXED POINT THEOREM IN PROBABLISTIC METRIC SPACES

R. C. Dimri and U. C. Gairola

Department of Mathematics, HNB Garhwal University, Srinagar, Pauri Campus, Pauri Garhwal

In paper we introduce the concept of R-weak commutativity for a pair of selfmap in probablistic metric space and establish a fixed point theorem. Our result is a generalization of many well known previous results.

D:15

MATRIX TRANSFORMATIONS ON SEQUENCE SPACE l_π

K. Chandrasekhara Rao and N. Subramanian

Department of Mathematics, Alagappa University, Karaikudl

Let (π_k) be a sequence of positive numbers. Let $l_\pi^{\infty v}$ be the space of all complex sequence (x_k) such that $\left(\frac{x_k}{\pi_k}\right) \in l$. For each fixed positive integer k, s^k denotes the sequence $(0, 0, \ldots, \pi_k, 0, \ldots)$, π_k in the k-th place and zero at other places. First we show that the set $E=\{s^k : k = 1,2,\ldots\}$ is a

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determining set for l_π. The set of all infinite matrices transforming l_π into a *BK*- space Y is denoted by $(l_\pi : Y)$. We characterizes the classes $(l_\pi : Y)$ when $Y = l_\infty$, c_0, c, l^p $(p \geq 1)$, bv, bv_0, bs, cs, l_p. In summary we have the following table

$\nearrow$	l_∞	c_0	c	l^p $1 \leq p < \infty$	bv	bv_0	bs	cs	l_p
l_π	Necessary and sufficient condition on the matrix are obtained.								

D:16

COINCIDENCE AND FIXED POINT THEOREMS FOR A FAMILY OF MAPPINGS ON MENGER SPACES

Praveen Kumar Shrivastava, N. P. S. Bawa and Vivek V. Raich

Govt. P. G. College, Balaghat, Govt. College, Rewa 481 001

Using an idea developed by M. Stojakovic, we generalize some results of Singh and Pant.

D:17

SOME RESULTS IN FIXED POINT THEORY AND BEST APPROXIMATIONS

Kanhaiya Jha

Kathmandu University, Kathmandu

D:18

FIXED POINT ITERATION PROCEDURES

Mahesh Chandra

G. B. Pant University of Agri. & Tech., Pantnagar 263 145

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D:19

PARTITIONS OF UNITY AND APPLICATIONS

Pramila Srivastava

10, C.S.P. Singh Marg, Allahabad 211 001

D:20

STABILITY OF ITERATIVE PROCEDURES FOR BANACH TYPE OPERATORS

P. Pradhan and S. L. Singh

Department of Mathematics, GKV, Hardwar 249 404

The intent of this note is to present stability theorems for Banach type operators in *b*-metric spaces. Our results extend a known similar results in metric spaces.

D:21

BEST SIMULTANEOUS APPROXIMATION AND SOME GENERALIZED NORMS OF *TYPE A* AND *TYPE M*

Vinai Kumar Singh and R. N. Mukherjee

Department of Applied Mathematics, Institue of Technology, Banaras Hindu University, Varanasi 221 005

In the present paper we have dealt with some of the problems in best simultaneous approximation under some generalized *type A* and *type M* norm conditions and their applications. Some generalized orthogonality conditions of the type introduced recently by Dragomir have also been used in the process. Our results in some way extend; some of the earlier results obtained in the same direction by Benitez and Fernandez.

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D:22

AN OLD PROBLEM OF SAMUELSON AND THE JACOBIAN CONJECTURE

T. Parthsarathy

Indian Statistical Institute, New Delhi 110 016

Suppose F is a C^1 - differentiable map from $\Omega \subset R^n \to R^n$, what condition should we impose on Ω and F such that F is (globally) one-to-one on Ω. Let J stand for the Jacobian of the map F. Non-vanishing of J over Ω need not imply one-to-one of F in general. This problem arises naturally in Mathematical Economics in connection with "Factor price equalization" considered by the well-known economist and Nobel-Lawreate, Paul Samuelson. We will discuss this problem in some depth and relate it to another well-known conjecture called "The Jacobian Conjecture"

D:23

EXTENSION OF RHOADE'S STABILITY RESULT

Virendra Arora and Mohini Pundir

Department of Mathematics and Statistics, GKV, Hardwar 249 404

The intent of this paper is to extend a few stability result of B. E. Rhoades to b-metric spaces.

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E:1

EXISTENCE OF SOLUTIONS OF SECOND ORDER SEMILINEAR NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS

S. Marshal Anthoni and K. Balachandran

Department of Mathematics, Bharathiar University, Coimbatore 641 046

In this paper we prove the existence of solutions for semilinear second order neutral functional differential equations using the theory of strongly continuous cosine families. The result are obtained by using the Schaefer theorem.

E:2

EXISTENCE OF SOLUTIONS OF NEUTRAL FUNCTIONAL INTEGRODIFFERENTIAL EQUATION IN BANACH SPACES

R. Sakthivel and K. Balachandran

Department of Mathematics, Bharathiar University, Coimbatore 641 046

In this paper we prove the existence of solutions for neutral functional integrodifferential equation in a Banach space. The result are obtained by using the Schaefer fixed point theorem.

E:3

ON SOME THEOREMS RELATED TO A SYSTEM OF SIMULTANEOUS DIFFERENTIAL EQUATIONS

S. N. Singh and U. S. Metha

Department of Mathematics, V.K.S. University, Ara

In this paper some theorems regarding Wronskian of the solutions associated with a system of simultaneous differential equations have been proved.

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E:4

NONOSCILLATION OF NONLINEAR NEUTRAL DELAY DIFFERENTIAL EQUATION

S. C. Palaniswami and E. K. Ramasami

Department of Mathematics, Bharathiar University Coimbatore - 641 646

Some sufficient conditions for the existence of nonoscillatory solutions of a nonlinear neutral differential equation are obtained by using fixed point technique. An example is provided to illustrate the theory.

E:5

A SECOND ORDER DIFFERENTIAL SYSTEM WITH NON SMOOTH COEFFICIENTS

P. K. Sengupta and Manika Sengupta

Department of Pure Mathematics, Calcutta University, Calcutta 700 019

We consider the Dirichlet problem

$$\mathcal{L}\phi=0,\ \phi(0) = \phi(1) = 0$$

assuming that it possesses only the trivial solution,

where

$$\mathcal{L} \equiv \begin{pmatrix} -\frac{d}{dx}\left(p_1\frac{d}{dx}+p_2\right)+q_1\frac{d}{dx}+r_1 & q_2\frac{d}{dx}+r_2 \\ q_2\frac{d}{dx}+r_2 & -\frac{d}{dx}\,p_3\,\frac{d}{dx}+p_4\,)+\,q_3\frac{d}{dx}+r_3 \end{pmatrix}$$

$\phi = (u, v)^T$, $p_1, p_2 \in L^\infty(0,1)$, $p_2, p_4, q_1, q_2, q_3 \in L^2(0,1)$, $r_1, r_2, r_3 \in L^1(0,1)$, $p_1, p_3 \geq \alpha > 0$ almost everywhere on (0,1); $\alpha \equiv$ constant and show that the boundary value problem

$$\mathcal{L}\phi = f = g,$$
$$\phi(0) = \phi(1) = 0$$

has a unique solution ϕ belonging to the sobolev space $H_0^1(0,1)$.

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E:6

GLOBAL EXISTENCE OF SOLUTIONS OF NONLINEAR FUNCTIONAL INTEGRAL EQUATIONS

M. B. Dhakne

Department of Mathematics, Dr. B. A. Marathwada University, Aurangabad 431 004

In the present paper global existence resutls for certain nonlinear functional integral and integrodifferential equations are established. Our analysis is based on an application of the Leray-Schauder alternative.

E:7

EXISTENCE OF SOLUTIONS OF QUASILINEAR INTEGRO-DIFFERENTIAL EQUATION WITH DEVIATING ARGUMENTS

K. Balachandran

Department of Mathematics, Bharathiar Univesity, Coimbatore 641 046

In this paper we prove the existence of mild solution for quasilinear integrodifferential equation with nonlocal condition. The results are established by the method of compact semigroups and the Schaefer theorem.

E:8

EXISTENCE OF WEAK POSITIVE SOLUTION OF COUPLED HAMMERSTEIN INTEGRAL EQUATIONS VIA FIXED POINT TECHNIQUE

B. C. Dhage

Department of Mathematics, Mahatma Gandhi College, Ahmedpur Lature 415 515

In this paper a coupled fixed point theorem concerning the solution of the operator equations $\lambda x=T(x,y)$, $\lambda y=T(y,x)$ is proved in Banach spaces which is further applied to the coupled Hammerstein integral equations.

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$$\lambda x=(t) = \int_J k(t,s)\, f(s,x(s),y(s))\, ds$$

and $$\lambda t=(t) = \int k(t,s)\, f(s,y(s), x(s))\, ds$$

for proving the existence of the coupled positive solution under suitable conditions. To illustrate the abstract theory developed here, an application is given to the nonlinear coupled boundary value problems of ordinary differential equations for proving the existence of coupled positive solution.

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F:1

SOME THEOREMS CONCERNING PSEUDO ANALYTIC VECTORS ON PSEUDO-KAEHLERIAN MAINIFOLDS

A. K. Singh and Sushil Rana

Department of Mathematics, HNB Garhwal University, Campus Tehri, Tehri-Garhwal 249 001

In the present paper, we have defined pseudo-analyticity and have investigated some theorems which concern pseudo-analytic vectors on pseudo-Kaehlerian manifolds.

F:2

ON GENERALISED RECURRENT RIEMANNIAN MANIFOLDS

Hukum Singh and Kuddus Khan

Department of Education in Science and Mathematics, NCERT, New Delhi 110 016

In this paper we have studied on a generalized recurrent Riemannian manifold using two 1-forms A and B. It is shown that for a non-zero constant scalar curvature, A is closed if B is closed for a non-constant scalar curvature the 1-forms A and B can not be closed both, unless A is collinear with B. Also, we have found out some results on a generalised conformally recurrent Riemannian manifold.

F:3

ON AFFINE CONNEXIONS IN AN ALMOST HYPERBOLIC CONTACT MANIFOLD

Savitri Verma

Department of Mathematics, Govt. P.G. College, Rishikesh 249 201

In this paper we define several affine connexions in an almost hyperbolic contact manifold and study some of its properties.

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F:4

ON COSYMPLECTIC MANIFOLD WITH CONTACT BOCHNER CURVATURE TENSOR

S. N. Pandey and Savita Verma

Department of Mathematics, Govt. P.G. College, Rishikesh 249 201

In this paper we have duscussed Cosymplectic manifold M_n (n=2m+1), possessing the orthonormal basis $\{e_1, e_2, e_3, \text{----------}, e_{2m}, Fe_1, Fe_2, Fe_3, \text{------}, Fe_{2m}\}$ of unit vectors which are normal to the contact vector. If we have obtained the expressions relating the sectional curvature and the scalar curvature in this manifold with vanishing contact Bochner curvature tensor.

F:5

ERROR CORRECTING CODES, MATROIDS AND ALGEBRAIC GEOMETRY

Sudhir R. Ghorpade

Department of Mathematics, IIT, Bombay, Powai, Mumbai 400 076

We shall discuss a basic problem in the theory of error correcting codes, which admits equivalent formulations in terms of uniform matroids as well as Grassmannians over finite fields. After reviewing the earlier work on this problem, we shall describe some recent joint work with G. Lachaud. Applications to other aspects of Coding Theory as well as to Algebraic Geometry will also be discussed. An attempt will be made to keep the prerequisites at a minimum.

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G:1

COINCIDENCES AND FIXED POINTS OF NONEXPANSIVE TYPE MAPS

S. L. Singh, D. D. Sharma and S. S. P. Singh

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Fixed point theory of nonexpansive and nonexpansive type maps provides techniques for solving a variety of applied problems in mathematical sciences and engineering. The purpose of this paper is to study nonexpansive type maps studied by M. Chandra, S. N. Mishra, S. L. Singh and B. E. Rhoades [Indian J. Pure Appl. Math 26 (1995), 393-401] on the setting of 2-metric spaces.

G:2

A UNIFIED COINCIDENCE THEOREM

S. L. Singh, D. D. Sharma and S. S. P. Singh

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In this paper we shall obtain a general coincidence theorem for Matkowski type operators on a finite product of 2-metric spaces. Several coincidences and fixed point theorems are obtained as special cases of our theorem.

G:3

INTUITIONISTIC GRADATION OF OPENNESS : INTUITIONISTIC FUZZY TOPOLOGY

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Let X be a nonempty set. We define an intuitionistic gradation of openness of fuzzy sets of X as an ordered pair (τ,τ^*) of functions form I^X to I such that

(1) $\tau(\lambda)+\tau^*(\lambda)\leq 1, \forall \lambda \in I^X$,

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(2) $\tau(\tilde{0})+\tau(\tilde{1})=1;\ \tau^*(\tilde{0})=\tau^*(\tilde{1})=0,$

(3) $\tau(\lambda_1\cap\lambda_2) \geq \tau(\lambda_1)\wedge\tau(\lambda_2);\ \tau^*(\lambda_1\cap\lambda_2) \leq \tau^*(\lambda_1)\vee\tau^*(\lambda_2),$

(4) $\tau(\cup_i\lambda_i) \geq \wedge\ \tau_i(\lambda_i);\ \tau^*(\cup_i\lambda_i) \leq \vee_i\tau^*(\lambda_i).$

The triplet (X, τ, τ^*) is called an intuitionistic fuzzy topological space. With this definition of an intuitionistic fuzzy topological space, we prove that the category of intuitionistic fuzzy topological spaces and gradation preserving mappings from a topological category. We also study compactness of intuitionistic fuzzy topological spaces and prove Tychonoff theorem.

G:4

ON λ-CONTINUOUS MAPS IN TOPOLOGICAL SPACES

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Several researchers in topology have introduced several weaker forms of closed sets and continuous maps in topological spaces. In this paper, we introduce a new concept called "λ-closed set" which is defined as follows : *A* subset *A* of *a* topological space *X* is called λ-closed if *A* contains the closure of *G* whenever *A* contains *G* and *G* is open. We observe that λ-closed set is weaker than closed set. Using λ-closed set, we introduce λ-continuous maps. Some properties of the newly introduced concepts have been investigated.

G:5

ON GENERAL, HYBRID CONTRACTIONS

S. L. Singh and S. N. Mishra

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Khan *et al.* have studied the existence of solutions of functional equations $f_i x \in Sx \cap Tx$ and $x = f_i x \in Sx \cap Tx$ under certain nonlinear hybrid contractions and asymptotic regularity conditions, generalizing many known results on coincidences and fixed points. However, most of their

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main theorems admit counter examples. In this paper, we have rectified these results and have obtained many coincidence and fixed point theorems rather on a more general setting.

G:6

FIXED POINT THEOREMS FOR JUNGCK TYPE HYBRID CONTRACTIONS

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The purpose of this paper is to show the existence of a common fixed point for triplets of maps (P,Q,f) and (T,F,g) satisfying

$$\delta(Px, Qy) \leq q.\max\{d(fx, fy), H(fx, Px), H(fy, Qy), \tfrac{1}{2}[H(fx, Qy)+H(fy, Px)]\}$$

and $$\delta(Tx, Ty) \leq q.\max\{d(fx, gy), H(fx, Tx), H(gy, Ty), \tfrac{1}{2}[H(Fx, Ty)+H(gy, Tx)]\}$$

for all x, y in X, wherein (X,d) is a metric space, H is the Hausedorff metric induced by d, $\delta(A, B) = \sup\{d(x, y) : x \in A, y \in B\}$, $P, Q, T : X \to CB(X)$, $f, g : X \to X$, $0<q<1$ and $CB(X)$ is the collection of all nonempty closed and bounded subsets of X.

G:7

COINCIDENCE THEOREMS FOR FOUR MAPS

S. L. Singh and Shikha Gupta

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The purpose of this paper is to obtain coincidence theorems for four maps on an arbitrary set with values in a metric space. As a consequence, results of Singh *et al.* are obtained as special cases.

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G:8

MULTI-VALUED MAPPINGS WITH COMMON INVARIANT POINTS IN 2-METRIC SPACES

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The notion of 2-metric space was first introduced by S. Gähler in 1963/64 and since then it was extensively explored by among others, C. Diminnie, A. G. White, K. Iseki and B. E. Rhoades. In the sequal following Banach contraction principle Jr. S.B. Nadler introduced the concept of multi-valued contraction mapping and established that it possesses a fixed point in a complete 1-metric space. Later on Chang and Huang in 1989 proved some fixed point theorems for set-valued mappings in 2-metric spaces. This paper intends to generalize the corresponding results.

G:9

FIXED POINT OF EXPANSION MAPPINGS ON RANDOM 2-NORMAL SPACES

B. D. Pant

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In this paper, the concept of expansion mappings on a random 2-normal space has been introduced and some fixed point theorems for such mappings have been established. There results being new of their kind, are interesting enterious of some known results.

G:10

D-CONTINIOUS FUNCTIONS

Suresh Pal Kohli and V.K. Sharma

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In the present paper we investigated some theorem which concern D_i- continuity on general topological space.

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G:11

NONLINEAR CONTRACTION IN *D*-METRIC SPACES

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In this paper a fixed point theorem is proved for the self-maps f of a D-metric space X satisfying the nonlinear contraction condition, namely.

$$\rho^r(fx, fy, fz) \leq \phi\,(\rho^r(x, y, z) \qquad (*)$$

for all $x, y, z, \in X$, $r \in N$, where $\phi : R^+ \to R^+$ is a continuous nondecreasing function satisfying $\phi(t) < t$, $t>0$.

Our result generalizes the fixed point theorem of Dhage [Bull. Cal. Math. Soc. 84(4) (1992), 329-336] and Dhage and Rhoades [Epsilon Jorn. Math (accepted)].

G:12

TOPOLOGICAL QUANTUM FIELD THEORY

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H:1

MINIMUM ENTROPY PROBABILITY DISTRIBUTION

Anju Rani and J. N. Kapur

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An analytical expression for the minimum entropy *Smin* as a function of prescribed value m of any mean (arithmetic, geometric, harmonic, power) is obtained when the random variate takes values $x_1, x_2, \ldots\ldots, x_n$; where $0 < x_1 < x_2 < \ldots\ldots x_n$ It is shown that (1) in the ith subinterval (x_i, x_{i+1}) $i=1,2\ldots\ldots, n-1$; *Smin-m* curve consists of two arcs. One starting from x_i with zero value and increasing to a maximum value (*Smin*) *max* at yi $(x_i < y_i < x_{i+1})$ and other starting from the value (*Smin*) *max* at y_i and decreasing to a value zero at x_{i+1}.

(2) *Smin-m* curve is continuous throughout, but is not differentiable at $y_1, y_2, \ldots, y_{n-1}$.

(3) $$\left[\frac{Smin\text{-}m}{dm}\right]_{x_i+0} = \infty, \qquad \left[\frac{d\,Smin}{dm}\right]_{y_i-0} > 0,$$

$$\left[\frac{d\,Smin}{dm}\right]_{y_i-0} < 0, \qquad \left[\frac{d\,Smin}{dm}\right]_{x_{i+1}-0} > -\infty.$$

and $\left[\frac{d\,Smin}{dm}\right]$ does not vanish for any value of x.

(4) *Smin-m* curve has cusps at $x_2, x_3, \ldots\ldots, x_{n-1}$;.

(5) The n-1 values of (*Smin*) *max* in the intervals are equal if arithmetic (geometric, harmonic, r^{th} order power) mean is prescribed and $x_1, x_2, \ldots\ldots, x_n$; (or $\ln x_1, \ln x_2, \ldots\ldots \ln x_n$; $1/x_1, 1/x_2, \ldots\ldots, 1/x_n$; $x_1^r, \ldots\ldots, x_n^r$) are in arithmetic progression.

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I:1

ON THE RATE OF CONVERGENCE BY MODIFIED SZASZ-MIRAKYAN OPERATORS

Vijay Gupta and Prabhakar Gupta

Department of Mathematics, IET, MJP Rohilkhand University, Bareilly

Recently Gupta and Srivastava (1995) introduced a new kind of Szasz-Mirakyan operators by taking the weight functions of Baskakov operators, so as to approximate Lebesgue integrable functions on $[0,\infty)$ interval. In the present paper, we extend the result of Gupta and Srivastava, we obtain a Voronovskaja type asymptotic formula and an estimate of error for the linear combinations of these summation-integral type operators.

I:2

RATE OF CONVERGENCE OF MODIFIED BASKAKOV TYPE OPERATORS OF FUNCTIONS OF BOUNDED VARIATION

Prabhakar Gupta

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Recently Aniol and P. Pych Taberska [Approximation Theory and its Appl. 11:2 (1995), 94-105] obtained the rate of pointwise convergence of some Durrmeyer type summation - integral operators. Gupta and Kumar [Demonstratio Mathematica *XXX* No.2 (1997) 339-346] used some results of probability theory and they estimated the rate of convergence for Baskakov-Durrmeyer operators of functions of bounded variation. In the present paperwe obtain the rate of pointwise convergence for recently introduced integrated Baskakov type operators, We have improved the earlier results of Guo and Khan [J. Approx. Theory 58 (1989), 90-101] and Aniol and Taberska (1995).

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I:3

ON THE INTERPOLATION SERIES OF ENTIRE FUNCTIONS

Devendra Kumar

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The paper deals with coefficients characterization of lower (p, q)-order and generalized lower (p, q) - type of Newton's interpolating series of entire functions. Finally, some inequalities connecting generalized (p, q)- type and generalized lower (p, q)-type fo interpolating series have been drived.

I:4

RELIABILITY TREES-A NEW CONCEPT FOR PERFORMANCE EVALUATION OF A COMPUTER COMUNICATION NETWORK

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Overall reliability of a Computer Communication Network (*CCN*) is one of the most important criteria for the determination of its performance. In case of a *CCN* having all duplex links, the concept of the spanning trees formed the basis of most of the researches carried out for the evaluation of network overall reliability. the conventional definition of spanning trees for *CCNs* involving some (or all) simplex links given as "A set of links which is free from cycles and connects all the nodes of the network", does not provide both way communication paths between each pair of nodes of the network. Therefore, no research has so far been carried out for evaluating overall reliability of a *CCN* involving simplex links. The aim of this paper is to introduce a new concept of reliability tree (RT) for evaluating overall reliability of a *CCN* having some oriented channels. Enough examples have been included in the body of the paper to explain the concept.

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I:5

ON THE DEGREE OF APPROXIMATION BY SPECIAL POSITIVE LINEAR OPERATORS

A. K. Sao, R. K. Gouraha, A. S. Ranadive, P. Verma and S. P. Singh

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In the present paper, some approximation properties on a special positive linear operators defined as :

$$(L_n f)(x) = \frac{1}{\varnothing_n(x)} \sum_{k=0}^{\infty} a_n, k\, x^k \frac{n^{\lambda+k+1}}{\underline{|\lambda+k+1}} \int_0^{\infty} e^{-nt}\, t^{\lambda+k}\, f(t)\, dt$$

have been discussed. (Here $o_n(x)$; a_n, k are some suitable choices of functions defined in the paper).

I:6

LEE DISTANCE AND NEGACYCLIC CODES

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The properties of the BCH (Base-Chavdhuri- HOCQuenghem) codes have been extensively investigated. Negacyclic code is a form of BCH code and utilises the concept of Lee distance whose properties have been stated.

I:7

DECIDING LINK PRIORITIES IN THE DESIGN OF A COMPUTER COMMUNICATION NETWORK FOR TERMINAL RELIABILITY OPTIMIZATION

Vinod Kumar, K. K. Aggarwal, M. S. Aswal

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Building reliability into the design of a Computer Communication Network (*CCN*) is the need of the hour which depends on the reliabilities

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of the network elements and its topological design. The topological design is basically concerned with location of computers and communication links connecting them so as to satisfy the performance requirements with the least cost. The problem of preparing a topological layout of a *CCN* with optimum *ST*-reliability is faced by the network designer whenever a new network is set up or an old network is expanded. The present paper attempts to address the above problem by suggesting a heuristic approach for the same. The goal is achieved by deciding the physical location of computers and communication links in such a way that the network required for desired connectivity is optimized. Thus the *ST*-reliability of a *CCN* with a fixed topology is optimized for a given set of link-reliability of a *CCN* with a fixed topology is optimized for a given set of link-reliabilities. The results reported in this paper have been compared with those derived from exhaustive enumeration technique. It is observed that these results are very close to the true optimum results in all the cases.

I:8

ON L_1-APPROXIMATION BY THE GENERALISED SZÃSZ - MIRAKJAN OPERATOR

D. R. Sahu, P. Verma and S. P. Singh

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The object of the paper to give the degree of L_1 approximation to an integrable function f by the generalised Szãsz Mirakjan operator defined as

$$(L^*_{n+p}f) = (n+p)\sum_{k=0}^{\infty} e^{-(n+p)} \times \frac{(n+p)^k x^k}{1.2.....k} \int_{k/n}^{(k+1)/n} f(t)\,dt$$

for $n+p \geq 0, p \geq 0, f \in L_1[0,\infty)$

involving the integral modulus of continuity $\omega(f;\delta)$ in L_1 norms with the density function $\sqrt{x}$.

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I:9

RESPONSE TIME ANALYSIS OF PAKET SWITCHED NETWORK

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For transmission of data beyond a local area, communication is tyorically achieved by transmitting data from source to destination through a network of intermediate switching nodes. The switching nodes are not concerned with the content of the data, rather their purpose is to provide a switching facility that will move the data from node to node as it reaches its destination.

In the present paper the development of an appropriate queuing model is presented for the performance evaluation of the packet switching in data communication. It consists of a transmitter and a receiver connected together with a communication channel in a packet switched network. A pair switching of nodes can be considered as combination of a transmitter and a receiver. Arrival of the message at the transmitter is assumed to follow Poisson distribution. Main aspect of the model is priority service schemes of transmitters. Two types of messages are assumed with arrival rates λ_1 and λ_2 respectively coming to a transmitter. If λ_1 message has priority over λ_2 then transmitter transmits first λ_1 then λ_2 .The result for mean waiting time at the transmitter, mean transmission time & mean service time of receiver have been derived to determine the mean response time of the complete system. The result are depicted in both tabular as well as in graphical forms.

I:10

STRONG UNIQUENESS OF BEST SIMULTANEOUS APPROXIMATION

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For investigating strong uniqueness of best simultaneous

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approximants, one considers here three properties of the triples $(X,V,\mathcal{F})$ where X is a normed linear space, V is a nonempty closed subset of X and $\mathcal{F}$ is a family of nonempty closed and bounded subsets of X. These properties ae called *SUBSA*, α-*SUBSA* and *SUBSA* of order q. Examples satisfying these properties are explored. In particular, the classical Haar theory for $C_0(T)$ as well as the more recent Haar theory for the function space $C_0(T,H)$ of Hilbert space-valued continuous functions are extended to best simultaneous approximants of certain sets.

I:11

OPTIMIZATION MODEL FOR RELIABILITY INDEX BASED TASK ALLOCATION IN DISTRIBUTED SYSTEMS

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The reliable performance for a Computer Communication Network (CCN) under various conditions is of utmost importance in many industrial, military and everyday life situations. Many desirable services (e.g. file service, authentication service in distributed system need be dealt with highly fault - tolerant and secured manner. Therefore, it is desirable to increase the reliability and security of services by destributing the responsibility for providing the service among many servers. There are many ways to increase the performance of a distributed system. A common approach is to distribute the processes and computer resources in such a way that the overall execution time is optimized. The present paper aims at developing a reliability - based optimal task allocation model for distributed systems. First we suggest a reliability and cost based index, based upon which overall task allocation reliability is optimized. The developed algorithm is coded into *FORTAN*-90 and implemented on *RISC* based *DEC*-2000 machine. The time complexity of the algorithm is observed to be $O(n^m)$.

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I:12

SOME APPROXIMATION RESULT ON A POSITIVE LINEAR OPERATORS

P. Verma and A. K. Sao, R. K. Gouraha, A. S. Ranadive and S. P. Singh
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Department of Mathematics, G.G.D. University, Bilaspur

In the present paper we define a sequence of positive linear operators as :

$$(L_n f)(x) = \frac{e^{-nx}}{g(1)} \sum_{k=0}^{\infty} p_k(nx) \frac{n}{dk(1)} \int_0^{\infty} p_k(nt)\, e^{-nt} (S_n f)(t)\, dt \qquad 1.1$$

where $(S_n f)(t) = e^{-nt} \sum_{k=0}^{\infty} \frac{(nt)^k}{\lfloor k} f(\frac{k}{n})$ and $p_k(nx)$ and $dk(x)$

are some suitable functions defined in the paper. We prove some approximation results on the operators and deduce some result on the corresponding operators also.

I:13

SOME APPROXIMATION PROPERTIES ON A BASAKOVA TYPE OPERATOR

A. K. Sao, R. K. Gouraha, A. S. Ranadive, P. Verma and S. P. Singh
Department of Mathematics, Guru Ghasi Das University, Bilaspur

In the present paper we define a sequence of a Basakova type operators, approximating a function $f \in L_1[c,\infty)$ as,

$$(L_n f)(x) = \sum_{k=0}^{\infty} \frac{(-x)^k \phi_n(x)}{\lfloor k} \frac{n}{dk(t)} \int_0^{\infty} e^{-nt} p_k(nt) f(t)\, dt$$

where $\phi_n(x)$ and $dk(t)$ are some suitable choice of functions defind in the paper. We prove some approximation results on the operators and deduce some results on the corresponding special operators also.

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I:14

ON THE DEGREE OF L_1-APPROXIMATION BY THE METHOD OF INTEGRAL BASKAKOV OPERATORS

D. R. Sahu, P. Verma and S. P. Singh

Department of Mathematics, Guru Ghasi Das University, Bilaspur

The object of the paper to give the degree of L_1 approximation to an integrable function f by the method of integrable Baskakaov operators

$$(L_n f)(x) = n\sum_{k=o}^{\infty}\binom{n+k}{k}\frac{x^k}{(k+x)^{n+k+1}}\int_{k/n}^{k+1/n} f(t)\,dt$$

involving the integral modulus of continuity $\omega(f;\,\delta)$ in L_1 norms with the density function $\sqrt{x(1+x)}$.

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J:1

NONDOMINATED SOLUTIONS OF A SITE LOCATION PROBLEM WITH TWO OBJECTIVE WITHOUT ASSIGNMENT OF PRIORITIES TO THEM

Satya Prakash

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The problem of selecting upto a fixed number of sites from among a given number of potential facility sites and asigning customers to sites in such a way that each customer is assigned to a unique selected site, is considered. The problem has two objective without assignment of priorities to them. One objective is to minimize the sum of the distances and the other objective is to minimize the maximum distance among the distances from each customer to his/her assigned site. A heuristic method is developed to find the set of nondominated solutions of the bicriterion site location problem.

J:2

A FUZZY CONTROL SYSTEM FOR TRAFFIC LIGHTS MANAGEMENT

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Fuzzy logic has found widespread applications in many diverse fields including artificial intelligence, engineering, operations research, robotics and pattern recognition. In situations, where information required for taking decisions is not complete or its reliability questionable. Fuzzy logic has proved immensely beneficial. In this paper a Fuzzy Control System has been applied to manage Traffic Lights by demarcating the traffic density into various fuzzy categories such as Light, Medium and Heavy for an inference engine to analyse the output provided by the Control System to take necessary appropriate actions. This program can

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be used in situations that involve similar kinds of fuzziness in general and to traffic management in particular. The application to this program will ensure smooth flow of traffic thereby helping in the day to day management of vehicular traffic.

J:3

AN INVENTORY MODEL FOR BANKING SECTOR TO CONTROL CREDIT

R. M. Bhandari and P. K. Sharma

Department of Mathematics, D.A.V. (P.G.) College, Dehradun

When Central Bank reduces the credit rate of interest then Commercial Bank might place a large order and offer a reduced credit rate of interest to customers. An inventory model is developed to control credit in banking industry. The optimal order quantity and profit are determined for commercial Banks. The monetory phenomenon in banking sector is explained by sensitivity analysis results.

J:4

A SINGLE PERIOD STOCHASTIC INVENTORY MODEL WITH ERLANG DEMAND DISTRIBUTION

R. M. Bhandari

Department of Mathematics, D.A.V. (P.G.) College, Dehradun

A single period inventory problem with Erlang demand distribution is considered. The model contains the usual ingredients of a newsboy problem. Sale price, purchase cost, salvage value and shortage cost. In addition, there is a cost per unit of marketing effort, and the demand distribution is stochastically increasing in that effort. It is assumed that the distribution of demand can be shifted up by an increase in sales effort. The paper deals with the simultaneous determination of optimal order quantity and sales effort. The optimum profit associated with optimum marketing effort is also determined.

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J:5

ANALYSIS OF A PARALLEL TRANSIT SYSTEM UNDER HEAD OF LINE REPAIR PRIORITY

P. P. Gupta and Deepika

Department of Mathematics, P.G. College, Modinagar

This paper considers a system consisting of two subsystems connected in series with a single repair facility. One subsystem is j out of M : G system consisting of M identical units, while the other consists of N non-identical units arranged in series. The life times of the active units depends on each other in having simultaneous failure of all the operating units and repair times are distributed quite generally. The system breaks down completely if more than M-j+1 units in the parallel group are simultaneously in the failed states or if any failure occurs in the series group of if any failure occures due to critical human error. The availability and reliability function of the system are obtained simultaneously. Explicit expressions for the steady state availability of the system and the mean time to the first system failure are obtained.

J:6

SYMETRIC DUALITY IN VARIATIONAL PROBLEMS WITH *F*-CONVEXITY

S. K. Mishra

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Under F-convexity, F-concavity F-pseudoconvexity, F-pseudoconcavity, appropriate duality results are established for Wolfe and Mond-Weir type symmetric dual variational problems. Using these results duality theorems for fractional variational problems are also obtained under additional condition on the kernel function above pair of problems are exibited to be selfdual.

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J:7

QUEUEING NETWORK MODEL FOR MULTIPROGRAMING

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In this paper we present two different models for multiprogrammed computer system. The first takes advantage of the original Gordon and Newell closed queueing network model and therefore requires the use of exponential service times throught. The second model makes use of diffusion approximation which permits the use of arbitrary distributions but yields only approximate answers.

J:8

RELIABILITY PARAMETERS OF A COMPLEX SYSTEM UNDER WAITING AND MIXED FAILURE PATTERN

P. P. Gupta and Ekata

Department of Mathematics, M.M. (P.G.) College, Modinagar

The mathematical model under consideration consisting of n-units in series suffers multitypes of failures. The system has three modes; viz. normal, degraded & failed with hardware and human failure. All types of failure and waiting rates are exponential while repair rates are distributed quite generally. System state probabilities and other parameters are developed for the proposed model using supplementary variable technique.

J:9

DISTRIBUTED SERVICE NETWORKS : SEPARATE DISPATCHING POLICIES

Arif Nadeem

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This paper exhibits two distinct service network, and develops a

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dispatching policy for each under no cooperation conditions. These policies will serve as a basis for comparision with cooperation policies presented in the paper. The networks that have been chosen are very simple in order to clarify the exposition; however, an expansion to a more realistic case is quite straight forward.

J:10

MODEL FOR SINGLE LEVEL LOTSIZING AND SCHEDULING

Arif Nadeem and Amit Kumar Awasthi

Department of Mathematics, Bareilly College, Bareilly 243 005

In the short term planning the sequence of the production quantities is required. This paper deals with models where the size and the sequence of lots are datermined simultaneously. These include the models for single-level, multi-item, dynamic, capacitated lotsizing and scheduling of the discrete and continuous set up respectively. Although these types of models are based on different fundamental assumption, each of them is applicable for short-term scheduling with periods as days or shifts.

J:11

A MATHEMATICAL MODEL FOR JOB SHOP SCHEDULING PROBLEMS

Arif Nadeem and Saurabh Srivastava

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This paper investigates the application of the algorithms available for solving the job shop scheduling problem. We determine the most powerful solution procedure for n jobs m machines problem. Our results show that the algorithms presented in this paper are either comparable to or better than the best algorithms for job shop scheduling problems. Most importantly, they represent a novel approach to solution improvement which holds promise in many job shop scheduling problems.

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K:1

DIFFUSION OF CHEMICALLY REACTIVE SPECIES IN VISCO-ELASTIC FLUID FROM A STRETCHING SHEET

Ravi. M. Sonath and M. Subhas Abel

Department of Mathematics, Gulbarga University, Gulbarga 585 106

This paper presents the solution of diffusion of chemically reactive species in the laminar boundary layer flow over an elastic plane surface in a visco-elastic fluid. The visco-elastic flow is driven soley by the linearly stretched surface and the reactive species is emitted from this sheet and undergoes an isothermal and homogeneous one-stage reaction as it diffuses into the surrounding fluid. Asimlarty transformation is devised, which reduces the non-linear concentration conservation equation to ordinary differtntial equation.

K:2

APPLICATIONS OF FUZZY LOGIC TO CLOSED LOOP

Sanjay Singh

Department of Mathematics, Dayal Bagh University, Agra

This paper opens with some fundamentals of fuzzy set and its extension to fuzzy logic. Further, how one can be fit to be fuzzy and fuzzy model have been discussed. This is followed by a description and an example of a fuzzy controller for a steam turbine and a design methodology for a fuzzy system.

K:3

HEAT TRANSFER IN *MHD* VISCO-ELASTIC FLUID FLOW OVER A STRETCHING SURFACE

M. Subhash Abel, K. V. Prasad and Ambuja Joshi

Department of Mathematics, Gulbarga University, Gulbarga 585 106

In this paper we considered the study of heat transfer characteris-

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tics in *MHD* visco-elastic fluid over a stretching sheet with variable wall temperature and internal heat generation.

Two cases are considered (i) The sheet with prescribed surface temperature (*PST*) (ii) The sheet with prescribed wall heat flux (*PHF*)

The effect of visco-elasticity on heat transfer is to accelerate wall temperature gradient in both *PST* and *PHF* cases. The wall temperature gradient will have positive values for all values of prandtl number which is a reverse effect compared to *MHD* viscous fluid flow past a stretching sheet as discussed by Vajravelu and Rollins [8].

K:4

CRITICAL LEVELS FOR GRAVITY WAVES

P. Kandaswamy

Department of Mathematics, Bharathiar University, Coimbatore - 641046

Earlier studies on the critical levels for gravity wave propagation have established the possible existence of two type of critical levels.

i. The wave trappers which absorb waves that approach them irrespective of the direction in which the wave is approaching the critical level.

ii. The other type of critical levels, valve like critical levels, permit free transmission of waves across these levels without any attenuation while the same wave if approaches the critical level from the otherside of it is absorbed at the critical level.

The present study has established that first type of critical levels, the wave trappers are again valve like critical levels superimposed by a reflexion level. This in essence establishes that there exists only one type of critical level, that is, the valve like critical level for gravity wave propagation.

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K:5

ANHOLONOMIC DESCRIPTION OF TWO + PHASE *MFD* FLOWS IN A ROTATING COORDINATE SYSTEM

Ram Babu Mishra

Department of Mathematics, Banaras Hindu University, Varanasi 221 005

The equations governing the two-phase *MFD* flows has been written for steady incompressible flows. With one ignorable coordinate in orthogonal curvilinear coordinate system the solutions have been found out. The vortex-line triad has been employed to discuss the complex lamellar and Beltrami motions. It has been shown that for Beltrami two-phase magnetofluids with flat field restrictions, the Bernoulli surfaces contain the vortex lines if and only if the component of the viscous force along the vortex lines is equal to the component of force exerted by the particles on the fluid along the vortex lines.

K:6

TOWARDS QUASI ISOTROPIC STRESS-HEAT FLUX TURBULENCE MODEL FOR BUOYANCY DRIVEN FLOWS

A. K. Upadhyay and T. Dixit

Department of Mathematics, Govt. College, Rishikesh, Dehradun 249 201
Department of Mathematics, K.N. Govt. P.G. College, Gyanpur 227 304

The theoretical relation between the temperature-pressure gradient correlation and the pressure-strain correlation is discussed in context with the quasi isotropic turbulence model. It is shown that for such a field the modelling of the heat flux transport equation becomes consistent with the modelling of the Reynolds stresses transport eqations.

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K:7

EFFECTS OF THE DEPENDENT SURFACE INFILTRATION ON WATER TABLE VARIATION IN A SEMICONFINED AQUIFER : AN ALTERNATIVE ANALYTICAL SOLUTION

Basant Kumar Jha

Department of Mathematics, Faculty of Science, B.H.U., Varanasi-221005

An analytical solution for transient water table variation in a semiconfined and finite aquifer with recharge from canals and time dependent surface infiltration is presented. The solution is obtained by Laplace transform technique as well as appropriate transformation. It is found that appropriate transformation approach is simpler then Laplace transform technique. Numerical values of water table variation are obtained by these methods, which are in close agreement.

K:8

NATURAL CONVECTION BETWEEN COAXIAL VERTICAL CYLINDERS PARTIALLY FILLED WITH A POROUS MATERIAL

Trishna Paul and A. K. Singh

Department of Mathematics, Banaras Hindu University, Varanasi 221 005

This paper presents laminar fully developed free convection flow between two coaxial vertical cylinders partially filled with a porous matrix when the cylinders are kept at different temperatures. The Brinkman-extended Darcy model is used is used to simulate momentum transfer in the porous region. The two regions are coupled by equating the velocity and by considering shear strees jump condition at the interface. The effect of Darcy number on the velocity throughout porous domain and adjustable coefficient in the stress jump condition at the interface is shown graphically. It is observed that velocity is influenced by the shear stress jump condition at the interface.

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K:9

NATURAL CONVECTION DUE TO HEAT AND MASS TRANSFER IN A COMPOSITE SYSTEM

A. K. Singh, T. Paul and G. R. Thorpe

Department of Mathematics, Banaras Hindu University, Varanasi 221 005
Department of Civil and Building Engineering, Victoria University of Technology, Footscray, Victoria, Australia - 3011

A numerical study is performed to analyse heat and mass transfer phenomena due to natural convection in a composite cavity containing a fluid layer overlying a porous layer saturated with the same fluid. The flow in the porous region is modelled using Brinkman-Forchheimar-extended Daray Model that includes both the effect of macroscopic shear (Brinkman effect) and flow inertia (Forchheimer effect). The vertical walls of the two dimensional enclosure are isothermal whilst the horizontal walls are adiabatic. The two regions are coupled by equating the velocity and stress components at the interface. The resulting coupled equations in non-dimensional form are solved by an alternating direction implicit method by transforming them into parabolic form by the addition of false transient terms. The numerical results show that the amount of fluid penetration into the porous layer depends strongly upon the Darcy, thermal and solutal Rayleigh numbers. Average Nusselt number decreases while average Sherwood number increases with an increase of the Lewis number. The transfer of heat and mass on the heated wall near the interface depends strongly on the Darcy number.

K:10

RAYLEIGH-BENARD CONVECTION IN A SECOND ORDER FERROMAGNETIC FLUID WITH SECOND SOUND

Pradee. G. Siddheshwar

Department of Mathematics, Bangalore University, Central College Campus, Bangalore 560 001

The effects resulting from the substitution of the classical Fourier law by the non-classical Maxwell-Cattaneo law [1] in Rayleigh-Banard convection of a second-order ferromagnetic fluid are examined. The classi-

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cal approach predicts an infinite speed for the propogatin of heat. The non-classical theory involves a wave type heat transport (SECOND SOUND) and does not suffer from the physically unacceptable drawback of infinite heat propogatin speed. The principle of exchange of stability is not guaranteed in the problem unlike the case of the Newtonian analog with Second Sound. It is found that the results are noteworthy at short times and the critical eigenvalues are less than the classical ones.

K:11

THE STABILITY OF DOUBLE DIFFUSIVE CONVECTION IN AN INCLINED FLUID LAYER WITH AND WITHOUT INTERNAL HEAT AND MASS GENERATION

P. M. Balagondar

Department of Mathematics, Bangalore University,
Central College Campus, Bangalore 560 001

The linear stability theory is applied to examine the double diffusive convection that occurs in an inclined fluid layer confined between two parallel plates that are maintained at different temperatures and different concentrations. Using the normal mode approach that is slightly different, the amplitude functions are taken as functions of horizontal coordinate, we get a eighth order eigenvalue problem in which the differential equations involved are with variable cooffecients. The power series method is used to obtain the eigenvalue relation which can be solved numerically to obtain the exact stability conditions.

In the second problem, we obtain a similar eigenvalue problem of eighth order for double diffusive convection that occurs in an inclined fluid layer confined between parallel plates with uniformly distributed internal heat and concentration sources. It is assumed that the two plates bounding the layer are maintained at constant and equal temperatures. The power series metod is applied to obtain the equation for the eigenvalues. The stability conditions are obtained for various Prandtl numbers and Schmidt's numbers and various inclination angles. The stability sets in either as transverse travelling wave or as longitudinal stationary modes.

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We solve the problems, in the particular cases, of horizontal and vertical layers and for longitudinal rolls.

K:12

THE EFFECT OF CORIOLIS FORCE AND NON-UNIFORM TEMPERATURE GRADIENT ON LINEAR AND NONLINEAR CONVECTION IN ISOTROPIC AND ANISOTROPIC POROUS CHANNELS - AN ANALYTICAL STUDY

P. M. Balagondar

Department of Mathematics, Bangalore University, Central College Campus, Bangalore 560 001

The effects of coriolis force and non uniform temperature gradient on the onset of convection in a horizontal rectangular channel filled with an incompressible fluid saturated by anisotropic porous media is studied by means of linear stability analysis. We derive the critical Rayleigh number expression analytically, when the walls of the channels are assumed to be impermeable and heat conducting to establish a linear temperature distribution in the vertical direction. The solution of the problem is saught in the form of Fourier series with its coefficients as functions of horizontal co-ordinate. We found that the effect of rotation in anisotropic porous permeability and thermal diffusivity have a significant effect on the critical Rayleigh number and the study flow patterns and the effect of rotation more often destablises the system. We have also carried out the non-linear stability analysis based on the Stuart Watson weakly non linear approach and determine the essential non-linear correction to the critical Rayleigh number and the corresponding solution.

K:13

MODELLING OF ACOUSTIC WAVE PROPAGATION INSIDE IRREGULAR WAVEGULIDES

A. Kapur

Department of Mathematics, J.M.I. University, New Delhi 110 065

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K:14

BINGHAM FLUID FLOW IN AN INCLINED CHANNEL BOUNDED BY TWO PERMEABLE BEDS

P. V. Arunachalam, S. Sreenadh and P. Sankara Reddy

Dravidian University, Kuppam 517 425
Department of Mathematics, Sri Venkateswara University, Tirupati 517 502
Department of Mathematics, V.R. College, Nellore

The flow of a Bingham fluid in an inclined channel bounded by two permeable beds is investigated. The flow in the channel is according to Bingham model and the flow in the permeable beds is described by Darcy's law. The expressions for the velocity field, the pressure and the temperature distribution, the fractional increase in mass flow rate and the shear stress are obtained. The results are deduced and discussed.

K:15

VIBRATION OF NON HOMOGENEOUS INFINITE PLATE OF PARABOLICALLY VARYING THICKNESS CONSIDERING DAMPING EFFECT

Rajendra Kumar, D. C. Gupta and S. P. Kohli

Department of Mathematics, J.V. Jain (P.G.) College Sharanpur
Department of Mathematics and Statistics, GKV, Hardwar 249 404

In the present paper our investigations are devoted to the non homogeneous elastic bodies of prabolically varying thickness. The method used is frobenius for solving the problem.

K:16

TOWARDS QUASI ISOTROPIC STRESS-HEAT FLUX TURBULENCE MODEL FOR BUOYANCY DRIVEN FLOWS

T. Dixit, Anil Upadhyay and Dinesh Pratap Dubey

Department of Mathematics, K. N. Govt. P.G. College, Gyanpur 227304

The theoretical relation between the temperature-pressure gradient

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correlation and the pressure-strain correlation is discussed in context with the quasi isotropic turbulence model. It is shown that for such a field the modelling of the heat flux transport equation becomes consistent with the modelling of the Reynolds stresses transport equations.

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L:1

AXIALLY SYMMETRIC STATIONARY VACCUM FIELDS IN GENERAL RELATIVITY

M. D. Patel and R. M. Patel

Department of Mathematics, S.P. University, V.V.Nagar 388 120

A class of oxially symmetric stationary vacuum solutions which one of Petrov type N one obtained.

L:2

AXISYMMETRIC HYDROMAGNETIC STAGNATION FLOW OF A NEWTONIAN CONDUCTING FLUID

D. K. Phukan

Department of Mathematics, Moran College, Moranhat - 785 670

Axisymmetric hydromagnetic stagnation flow of a viscous, incompressible, electrically conducting fluid in the presence of a transverse magnetic fields is studied. A series solution of the similarity equation is presented for the case of large magnetic parameter. The Galerkin's approximate technique is used to solve the similarity equation expressed in terms of Crocco's variables. Then, an accurate expression the Shear Stress parameter is derived using Crocco's trnasformation. This is followed by a numerical solution of the similarity equation using a direct Runge-Kutta Shooting algorighm with Newton iteraction in double precision. Further more, the effects of the magnetic field on the velocity and Shear stress parameter, are analysed.

L:3

REMARKS ON A PAPER OF BANERJEE AND KATYAL

Harikishan and J. S. Parihar

Department of Mathematics, D. N. College, Meerut

Banerjee and Katyal (1988) established a result that, at the marginal state in the magnetohydrodynamic simple Benard problem, the total kinetic energy associated with a disturbance is greater than or equal to

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its total magnetic energy in the regime $Q\sigma_1 \leq 3$ which is not correct. Here we have obtained the correct form of this result.

L:4

ELECTRONS IN CONDUCTORS

V. N. Sharma and S. R. Chaudhry

Department of Mathematics, Govt. Degree College, Rudhauli Basti

This paper deals with the behaviour of electrons in conductors. Expression for the drift velocity is discussed. Individual and collactive roles are explained. For clear insight need of laser study is necessary.

L:5

POES TREATMENT OF DEPRECISSION

V. N. Sharma

Department of Mathematics, Govt. Degree College, Rudhauli, Basti

This paper deals with the poes treatment of deprecission.

L:6

MHD UNSTEADY FLOW OF A DUSTY VISCO-ELASTIC FLUID INDUCED BY THE MOTION OF A SEMI- INFINITE PLATE MOVING WITH VELOCITY DECREASING EXPONTIALLY WITH TIME

D. S. Negi

Department of Mathematics, H.N.B. Garhwal University Campus, Srinagar Garhwal 241 174

This paper concerns with the *MHD* flow of a dusty viscoelastic fluid induced by prescribed motion of a semi-infinite plate. Expressions are obtained for the fluid phase and the dust-phase velocity distributions and also for the magnetic field. Some interesting conclusions are made.

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L:7

COVARIENT FORMULATION OF SUPERFIELD LAGRANGIAN DENSITY IN "SUPER SPACE"

K. N. Baramola and D.C. Joshi

Department of Physics, Govt. P.G. College, Kotdwara

Extension of 3-dimensional Eucleadian space to 4-dimensional Minkowski space led to unify the electricity and magnetism into a single electromagnetic field strength tensor. Various attempts have been made to extend 4-dimensional space further to unify all the four fundamental interactions i.e. electromagnetic, gravitational, strong and weak interaction. Super-space formulation is a similar attempt in 4+4=8 dimensional space.

In this paper the structure of a super-field is discussed in super-space. In addition to this, super-symmetry and super-symmetric transformations of component fields are given and finally the Lagrangian formulation is carried out for scalar and vector super-fields.

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M:1

DYNAMICS OF HIV INFECTION OF *CD*4+*T* CELLS WITH TIME DELAY

Jonaki B. Ghosh

Department of Mathematics, Delhi University, New Delhi

We explore a model proposed by Alan S Perelson which describes the interaction of HIV with CD4+ T cells. The model comprises of four differential equations. Specifically four populations namely the uninfected T-cells, latently infected T-cells, actively infected T-cells and the free virus are considered in this model. It exhibits two kinds of steady states- the uninfected steady state where there is no virus present and the endemically infected steady state in which both virus and T- cells are present.

In this paper we propose a modified version of the model by introducing time lag. The model is based on the assumption that the interaction of the free virus and the T-cells has a delayed effect on all four populations. The terms in the differential equations of Perelson's model, indicating the interaction between the virus and T-cells are replaced by delay interaction terms with suitably chosen delay kernel function. Thus the differential equations of the model are replaced by intergrodifferential equations. we study the stability of both steady states by considering a distribured delay model and serveral combinations of delay kernel functions. An interestiong conclusion of the analysis is that an originally stable system remains stable after introducing time lag (irrespective of the choice of the delay kernel functions) whereas an originally unstable system may remain unstable or become stable depending upon the choice of the delay kernal function.

M:2

MATHEMATICAL MODEL FOR MASS TRANSFER IN AN ARTIFICIAL KIDNEY

V. K. Katiyar

Department of Mathematics, University of Roorkee, Roorkee

The object of the present investigation is to present analytical

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study of the mass transfer processes in a tabular hemodialyser. The dialysing fluid flows inside the tube and the blood flows over the tube in a thin film. The walls are supposed to be permeable. The attempt is made to estimate the redistribution of materials across the circular inter face and the resistance to the mass transfer in both the phases simultaneously. It has been shown that the application of metabolic reactions in the blood considerably intensify the concentration of diffusible components in the blood film.

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N:0

PRACTICAL APPLICATION OF VEDIC KNOWLEDGE IN TODAY'S MODERN WORLD

Dr. Pranav Pandya

Director, Brahmavarchas Research Institute, Shantikunj, Hardwar 249 411

N:1

REMARKS ON CERTAIN APPROXIMATION TECHNIQUES OF GANITATILKA

V. Arora and V. Goel

Department of Mathematics and Statistics GKV, Hardwar 249 404

The main purpose of this paper is to discuss and compare techuniques for solving equations $x^2=a$ and $x^3=a$ (a is any positive integer) as provided in *Ganitatilaka* and elsewhere. Also we have extended this technique to non-perfect and decimal numbers.

N:2

A FEW IMPORTANT APPLICATIONS OF FIRST DEGREE INDETERMINATE ANALYSIS IN HINDU ASTRONOMY

S. L. Singh and N. B. Singh

Department of Mathematics and Statistics, GKV and V.M.I.C., Hardwar

In this paper an attempt is made to discuss some applications of first degree Indeterminate Analysis in the field of Hindu Astronomy.

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study of the mass transfer processes in a tabular hemodialyser. The dialysing fluid flows inside the tube and the blood flows over the tube in a thin film. The walls are supposed to be permeable. The attempt is made to estimate the redistribution of materials across the circular inter face and the resistance to the mass transfer in both the phases simultaneously. It has been shown that the application of metabolic reactions in the blood considerably intensify the concentration of diffusible components in the blood film.

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N:0

PRACTICAL APPLICATION OF VEDIC KNOWLEDGE IN TODAY'S MODERN WORLD

Dr. Pranav Pandya

Director, Brahmavarchas Research Institute, Shantikunj, Hardwar 249 411

N:1

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N:3

INDIAN MATHEMATICS : A BRIEF HISTORIC EVOLUTION WITH REFERENCE TO IRRATIONAL NUMBERS

V. Mishra and S. L. Singh

Department of Mathematics, SLIET, Longowal and GKV, Hardwar 249 404

This paper presents a brief historical account of chronology of Vedic literature, elementary treatment of surd numbers, theorem of square on the diagonal, square and cube-rooting, some aspects on varga prakrti, values of pi, and tangent-and pi-series, Datta's semi-geometrical approach for obtaining $\sqrt{2}$ is an ingenious technique based on the lines with the prescription of the sulba geometry. This method paves for evaluation of this and other surd numbers in various ways, thereby giving rise to a number of series representation for $\sqrt{N}$ ($N>0$, a positive integer).

N:4

ON THE HISTORY OF ANCIENT INDIAN MATHEMATICS

K. K. Velukutty

Department of Mathematics, College of Arts & Science, Coimbatore

Using 'Kalpaganitha' and 'Irainar' Akaporul', Kesari Balakrishna Pilla fixed the Sangham age as 498-817 AD. The Keralite legendary, Parachipettapanthrirukulam is analysed and the members were identified as the members of Navarathnas of Kulasekara I. This author, continuing the same stream of thought, identified Aryabhatta and Baskara I as Vararuchi and Pakkanar of Panthirukulam. Kesari himself identified Haridatta as Naranathuranthan. The year and place of birth of Aryabhatta are fixed as 581 (instead of 746) and Padaliputra in Tamil Nadu (instead of Padaliputra in Bihar). On the way of asserthing the hypothesos, the principale of three phases and addresses of scholars is introduced : Aryabhatta-Kumarilabhatta-Mochikeeranar, Haridatta-Naranathubranthan-Ponmudiyar and Baskara I-Pakkanar-Philantholmoosathu.

Concludingly, exposing the uncertainity of periods of Deergathamasu, Meghadhithi, Bharadwaja and Varahamihira, the needs for chronological sequence of events in Puranic (3102-967 BC) and Ancient (967 BC - 1169

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AD) Indian Mathematics and the revival of finite mathematics due to Aryabhatta are stressed.

N:5

GEOMETRY AND ITS APPLICATIONS

Hukum Singh

Department of Education in Science and Mathematics, NCERT., New Delhi 110 016

The notion of geometry originated in ancient times. They arose out of the necessity to determine the lengths, areas, and volumes of various objects. About 4000 yeas ago rules for determining areas and volumes were compiled in Egypt and Babylonia. Egypt people used this knowledge in measuring land, areas. The Greeks took over from the Egyptians and Babylonians their geometric findings about 2500 years ago. The Greek scholars discovered various geometric properties which were used for the further development of the different concepts.

Geometry in todays knowledge means the branch of mathematics dealing with spatial figures. In ancient Greece all of mathematics was regarded as geometry. B. Pascal spoke about geometry as mathematical way of thinking.

There are several branches of geometry viz., Euclidean geometry, Descarte's geometry, Non-Euclidean geometry, Gauss and Riemannian geometry etc. In this paper we have discussed about all these geometries and their applications in physics, chemistry and Biology.

N:6

APPLICATION OF INDETERMINATE ANALYSIS IN HINDU ASTRONOMY

Ramesh Chand

Department of Mathematics, V.M.I.C. Sec.5, BHEL, Hardwar

In this paper an attempt is made to explain the application of indeterminate Analysis for the solution of Hindu Astronomical Problems.

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N:7

COMPUTATION OF ADHIKAMÃS BY CONTINUED FRACTION

Ramesh Chand

Department of Mathematics, V.M.I.C. Sec.5, BHEL, Hardwar

In this paper an attempt is made to explain the importance of Adhikamãs in Hindu Astronomy and computation of it by continued fraction.

N:8

GRAHALÃGHAVA AND COMPUTATION OF PLANETARY POSITION.

V. K. Sharma and Mustafizur Rahman

Department of Mathematics and Statistics, GKV, Hardwar-249404

In this paper an attempt is made to give the critical study of Grahalãghava and computation of planetary position, Tithi & Nakshatra etc, for any day.

N:9

A COMPARATIVE STUDY OF SIDEREAL REVOLUTIONS OF PLANETS IN A KALPA ACCORDING TO *BHÃSKARÃCHÃRYA* AND MODERN ASTRONOMY

V. K. Sharma and Mustafizur Rahman

Department of Mathematics and Statistics, GKV, Hardwar 249404

In this paper an attempt is made to give a comparative study of number of revolutions of planets in a Kalpa according to *Bhãskarãchãrya* and modern astronomy.

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N:10

RELEVENCE OF VEDIC MATHEMATICS AND ITS APPLICATION IN GLOBAL PRESPECTIVE

G. P. Pokhariyal

Department of Mathematics, University of Nairobi, P O Box 30197, Nairobi, Kenya

Vedas are the most ancient compositions of knowledge containing the recollections of sages, seers and saints through observations and meditation in *Aranyas* of physical nature and the FORCE behind it; and must be viewed as the treasure of entire manking. Mathematics is the branch of knowledge which is mainly based on logic, reasoning and truth that has DIVINE input. *Vedic mathematics* provides a module for viewing things of the creation in a unique Vedic tradition where rules, theorems and propositions are presented in very concise and short way, through use of Sanskrit. Well established scientific theories and the current findings in science can also be considered to converge to the *Vedic* views in unfolding the mystery of nature.

N:11

SIMILARITY OF PLANE FIGURES : A REFLECTION IN INDIAN GEOMETRY

Vinod Mishra and S. L. Singh

Department of Mathematics, SLIET, Longowal 148106
Department of Mathematics, GKV, Hardwar 249404

This *talk* attempts to highlight Indian knowledge as regards to similarity concept and its implemetation to various geometrical problems.

N:12

MATHEMATICS IN ANCIENT INDIA

B. S. Rajput

Kumaun University, Nainital 263 001

The purpose of this *talk* is to present a brief account of significant achievements of ancient Indian savants to mathematical sciences.

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N:13

VISVE-DEVA GODS-GROVR OF ANCIENT VEDIC PHILOSOPHY AND PHYSICAL DIMENSIONS OF MODERN SCIENTIFIC UNIVERSE

Rai Gyan Narain Prasad

Savitri Niwas (Behind Post Office) Mohaddipur Gorakhpur 273 008

In this paper we discuss dimensional relationship between the physical graduations and visvedevas.

N:14

BRIEF HISTORY OF DEVELOPMENT OF PROBABILITY

Vijayendra Kumar Sharma and Suresh Pal Kohli

Department of Mathematics and Statistics, GKV, Hardwar 249404

In this paper a brief account of development of probability theory through main contributions of Blaise Pascal, Jacoti Bermoulli, Abraham De Moivre, Baye, Gauss and Laplace etc is given.

N:15

VEDIC MATHEMATICS VERSUS MODERN MATHEMATICS WITH REGARD TO CERTAIN ASTRONOMICAL PHENOMENA

Shyama Charan Dubey

Vedic Research Institute, 51/102, Anand Nagar, Jalandhar 144 008

This *talk* intends to compare Vedic mathematics to with modern mathematics with reference to distance of sun from the earth, velocity of light, division of time and related issues.

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N:16

MATHEMATICS (GANIT) - A VEDIC STUDY

Manu Dev Bandhu

Ved Department, Gurukula Kangri Univeristy, Hardwar 249 404

The founder of Arya Samaj Swami Dayanand Saraswati proclaimed- वेद सब सत्य विद्याओं का पुस्तक है। वेद का पढ़ना-पढ़ाना और सुनना-सुनाना सब आर्यों का परम धर्म है। The Vedas are sources of all true knowledge. To know the rule of mathematical measurement and their relation is quite essential. This science of calculations and compilations primarily falls under the heading of *Athervaveda*. The *trishaptah* (त्रिषप्ताः) means triple seven viz., twenty one. There is a specific upveda on this subject, "Arthveda". which deals with this science.

सहस्रशीर्षा पुरुषः सहस्राक्षः सहस्रपात्।
स भूमिं सर्वतः स्पृत्वाऽत्यतिष्ठद्दशाङ्गुलम्।।(Yaju 31/1)

In this verse सहस्र (Sahasra) means innumerable dashangulam (दशाङ्गुलम्) means the world which is made up of ten parts namely, ten gross elements, and 5 subtle elements. It also means that God is manifested on all the ten sides.

पश्येम शरदः शतं जीवेम शरदः शतम् (Yaju 36/24)

We may be able to see for hundred years, we may live long for hundred years.

कतम आदित्या इति द्वादश वै मासाः संवत्सरस्य एते आदित्याः
(Shatpath brahman 3/9/5)

Who are there Aditya ? These twelve months of the year are known twelve Aditya. the rays of the Sun have different effects on each month, on us.

एकं सत् Truth (God) is one. (Rigved)

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N:17

FRACTIONAL CALCULUS : A PERSPICUITY OF ANALYSIS

P. K. Banerjee

Department of Mathematics, JNV University, Jodhpur 342 008

Knowing about the subject and the people who inspired since 1695, a houseful of great mathematicians, this is the story. By various amicable transformations, to day the subject of fractional calculus or differintegrals, has become an essential tool for applied mathematicians and for functional analysts. The text is prepared with a thought to introduce this branch of analysis to every one sitting in any corner of the globe : It contains basis, it has definitions and has various application sources. At this juncture of developments in mathematical analysis; I deem its presence extensively.

N:18

SUB-VEDA FOR CALCULATIONS

S. L. Singh

Department of Mathematics and Statistics, GKV, Hardwar 249 404

What is Vedic Mathematics ? Was there a separate *Veda* or *sub-Veda* for approximate and exact mathematical calculations ? This *talk* attempts to answer these questions.

N:19

NONLINEAR FLOW OF VEDIC MATHEMATICS

Virendra Arora

Department of Mathematics and Statistics, GKV, Hardwar 249 404

The intent of this *talk* is to discuss a few developmental aspects of mathematics of Vedic tradition.

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N:20

REMARKS ON SOME FIRE ALTARS OF *S'ULBAS*

Virendra Arora and Nidhi Handa

Department of Mathematics and Statistics, GKV, Hardwar 249 404

The purpose of this note is to compare of few fire altars of *S'ulba* tradition.

N:21

CALCULATION OF THE AGE OF UNIVERSE -ANCIENT INDIAN MYTHOLOGICAL VIEW

K. N. Baramola and S. C. Sharma

Department of Physics, Govt. P.G. College, Kotdwara

Age of the universe has been a topic of wide interest for those who wish to compute it. Above all the age of the universe shows the discord amongst the various workers. In this paper we have compared the age of universe by *Hubble's Law* with that given in the *Kalpant* as given in Indian Mythology. Surprisingly and coincidently it comes out to be equal. In addition to this, other methods of dating of univese e.g. radio active dating, galactic evolution dating method, stellar evolution dating method etc. have also been discussed and compared with the age calculated in Indian mythology which is unparalled in other religions.

N:22

METHOD OF DIVISION OF SECOND DEGREE POLYNOMIALS IN TWO OR THREE VARIABLES BY FIRST DEGREE POLYNOMIAL IN TWO OR THREE VARIABLES

S. M. Chauthaiwale

Anglo Hindi Junior College, Yavatmal 445 001

The Horner's method of synthetic division for division of polynomi-

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als in one variable (say x) is well known with the fact that this method is restricted to linear divisors of the type $ax+b$, a and b are constants.

In the book named 'VEDIC MATHEMATICS' author Swami Bharati Krishna Tirthaji has explained methods of division of polynomials by *Paravartya Sutra* where the restriction on degree of divisors is removed and divisor may be any lesser degree polynomial than dividend. No doubt this is generalization of Hoener's Method of Synthetic division, and current Mathematicians should reconize this attempt as one of the best contribution to mathematics.

This paper further extends Swamiji's method, to division of second degree polynomials in two or three variables. In this method divisor is modified by Paravartya Sutra, the dividend is arrenged in descending order of degree of terms in any one variable followed by terms in another variable/s in similar manner. The division is carried out, with some necessary adjustments, similar to Swamiji's method.

N:23

MATHEMATICAL DATA IN VEDAS

(Padmashri) Kapil Dev Dwivedi

Vishvabharti Anusandhan Parisad, Gyanpur (Varanasi) 221 304

This *talk* intends to survey mathematical compositions and references in Vedas.

N:24

LARGE NUMBERS IN VEDAS

Dinesh Chandra Shastri

Department of Veda, GKV, Hardwar 249 404

This note reports incidence of certain natural numbers in Vedas.

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Journal of Natural & Physical Sciences Vol. 12 (1998), No. 1, 124-126

IMPORTANCE OF INFORMATION TECHNOLOGY IN EDUCATION IN MODERN TIMES

R. Anand*

Telecommunication via geostationary satellites and the worldwide network of computers have made the world a global village. Real time electronic communication between any two computers connected to the network, irrespective of their spatial separation, can be realised using the internet. The internet has shrunk the world in both space and time and made the concept of national frontiers as national as the boundary lines drawn on a globe.

In a village, the education of children in addition to bring the prerogative of the parents is a community responsibility, because children grow up together. Likewise in the present context, education of children who now live in a global village may have to be viewed with a broader perspective then as isolated national concerns. In other words we can call "Growing Up Together". This will require that our children study in a global classroom. Internet offers the possibility to realise teaching and learning in a "global classroom".

The School Education System in India

India being a vast country is full of diversity not only in its social and cultural but also in its educational system. It is therefore not surprising that the structure of education in the country became extremely heterogeneous as it evolved in the States in response to aspirations and contextual needs of the regional population. Therefore, the most important task for the Central and the State Governments, which work together in a federal set up, was to adopt a common structure of education. Because of recent efforts, the country has finally adopted a uniform structure of education. The structure has been split into three stages, comprising 10+2+3 years of studies.

Today, the world has changed. More and more jobs have become skilled. The youth will have to possess the skills required for initial employment and the ability to learn new skills to keep pace with the

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changing nature of work. Another dimension to consider is that employment opportunities will be preferentially available to those who possess the required skills irrespective of their nationality. Therefore, young persons will require an education that can provide them better appreciation of life and culture in societies other than that of the society in which they grow up. The other crucial factor is the influence of information technology in nearly all occupations. Each person will have to learn how to use the computer as tool in the work that he/she may be doing and have the generic skills for using information technology effectively. Some of these skills are problem solving, critical thinking, decision making, accessing information, processing and dissemination of information after providing value addition to it.

The Students and the Teachers can take advantage of the vast educational resources available in the countries of the region, elsewhere in the world through the internet and other communication technologies. The personality of a child is shaped in a cultural context. By enabling young persons to grow up together in an interactive environment with peers in different countries will help them to attain comparable intellectual heights and develop international understanding. To make this happen we require participation of appropriate organisations in the region for working.

It may be pointed out that Educational Institutions in India emphasis is given on teaching of traditional subjects such as languages, mathematics, science and social sciences. What is required is that alongwith traditional subjects. They should be able to learn decision making skills, communication skills and interpersonal skills. These can be achieved by exposing students to real life problem, involving for their solution a collaborative effort. Implementation of such an approach will require that Educational Institutions, irrespective of their physical separation, arrange for their students to interact with one another to enable them to work on problems that can be best solved collectively. The internet provides an effective mode for enabling interaction in spite of physical separation. In addition, the global networking of computers has made if possible to shere educational resources whichout encumbrance and that too at near zero cost. Individuals and institutions are putting up information on every conceivable area of human interest on the Internet. The Internet is an ideal medium for interactive self-learning and if used intelligently with develop learning to learn. Therefore, the computer and the network to

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which it may be connected have to be used as tools for achieving the stated objectives and not as subjects of study.

It is expected that these competencies will the information technology will develop in them the confidence to upgrade their skills, as they would have acquired the ability of learning to learn.

The topics on which the Indian students should be able to carry out in-depth study and acquire information that they can share over the internet with their international peers, say students in Australia, are those that they have inherited, grown up with or have easy access to. For them, it would be a voyage into their own heritage and culture for discovering India. if similar approach is adopted by all the participating countries the children with their knowledge, understanding and experience will be able to mutually benefit and develop a realistic appreciation of each other's culture.

Role of Educational Institution

The Educational Institutions should take the responsibility of :

(a) Development of modules for students for acquiring the planned competencies
(b) Training of teachers
(c) Networking with professionals for providing expert support
(d) Obtaining feedback from students
(e) Making available on the internet its resources
(f) Answering students queries by e-mail

Looking ahead

It is accepted that a majority of schools in India have inadequate infrastructure and it may not be possible for the State to provide computers in sufficient numbers to each Educational Institutions for implementing the above programme. If with the pilot project, it can be convincingly shown that children who are information technology literates have an edge in an information society, the community may come forward to meet the needed support.

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Papers/Talks : Late Arrivals

T. T. Raghunathan : Enumeration of Bipartite Self Complementary Graphs.

M. M. Shikare : Splitting operation and Minors of Binary Matroids.

N. S. Bhave : Generalised Bipartite Complement of a Graph.

Y. S. Pawar : 0-1 Distributive Lattices and Complementedness.

S. K. Nimbhorkar : Theory of Rings and Lattices.

M. P. Wasadikar : Atomistic Semi-lattices.

Pornima Pawar : Approximation by Data Reduction.

M. A. Pathan : Lie Theoretic Approach to generalised Bessels functions.

P. Bhattacharya : Comlex dynamics during last 25 years.

Sundar Lal : Some Secret sharing schemes.

S. M. Bhave : Set Theory in Ancient Indian Logic.

V. K. Grutu : Historic Marginal Note and Fermat's Last Theorem.

K. N. Joshi : Realization of complex angular momentum operator.

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64th Indian Mathematical Society Conference and Symposia on Traditions of Vedic Mathematics and Applications & Fixed Point Theory in Nonlinear Analysis & Applications

December 19-22, 1998

Gurukula Kangri Vishwavidyalaya, Hardwar 249 404

DAILY PROGRAMME

DECEMBER 19, 1998

8.30 - 9.30	REGISTRATION
9.45 - 11.30	INAUGURAL SESSION
	Flag Hoisting & Procession
	Invocation
	Welcome Address by the Vice-Chancellor, GKV
	Report by the General Secretary
	Report by the Administrative Secretary
	Report by the Academic Secretary
	Inaugural Address by the Chief Guest
	Address by the Guest of Honour
	Presidential Address (General) by Prof. B. K. Lahiri
	Release of "Resonance of Ramajujan's Mathematics Vol. III : R. P. Agarwal": by the Chief Guest; Presentation by the General Secretary
	Award of P. L. Bhatnagar Memorial Prize to Shri Abhinav Kumar
	GKV Honours Mathematicians; Presentation by the Principal
	Vote of Thanks by the Local Secretary
	National Anthem
11.30 - 12.00	TEA
12.00 - 13.00	PRESIDENTIAL ADDRESS (Technical) : B. K. Lahiri Chairperson : R. P. Agarwal
13.00 - 14.30	GROUP PHOTOGRAPH FOLLOWED BY LUNCH
14.30 - 15.45	NINTH HANSRAJ GUPTA MEMORIAL AWARD LECTURE T. Parthasarathy : An Old Problem of Samuelson and the Jacobian Conjecture Chairperson : A. R. Singal

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15.45 - 16.00	TEA		
16.00 - 18.00	SPECIAL PAPER READING SESSION :		
	Presentation of papers for the Award of Prizes		
18.00	VISIT TO JAIRAM ASHRAM AND DINNER		

DECEMBER 20, 1998

09.30-10.00	INVITED TALKS	:	
	A.R. Singal	:	The Millenium Bug (The *Y2K* Problem)
	Chairperson	:	T. Parthasarathy
10.00-11.15	NINTH V. RAMASWAMY AIYER MEMORIAL AWARD LECTURE		
	V.M. Shah	:	A Peep Into the Grass Roots of All Mathematics Education
	Chairperson	:	M.K. Singal
11.15 - 11.30	TEA		
11.30 - 13.00	**SESSION - I**		
	SESSION ON LATTICE THEORY		
	Co-ordiantior	:	N.K. Thakare
	Speakers	:	Y.S. Pawar
			M.P. Wasadikar
			S.K. Nimbhorkar
11.30-13.00	**SESSION - II**		
	FIXED POINT THEORY SYMPOSIUM SESSION		
	Co-ordinators	:	B. K. Lahiri and S. L. Singh
	Inaugural Speaker	:	B.S. Yadav
	Other Speakers	:	P.P. Murthy, R.N. Mukherjaee, R.P. Pant, Geetha S Rao and others.
13.00 -14.00	LUNCH AT GURUKULA KANGRI PHARMACY		
14.00	EXCURSION TO RISHIKESH INCLUDING VISIT TO PRAMARTH NIKETAN		
17.00 - 18.30	SHRI GANGA ARATI, Blessings by Svami Chidanand Muni Ji and Prasadam		

DECEMBER 21, 1998

09.30 - 10.00	INVITED TALKS	:	
	S.B. Nimse	:	Topological Quantum Field Theory
	Chairperson	:	V.M. Shah

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10.00 - 11.15 TWELTH P.L. BHATNAGAR MEMORIAL AWARD LECTURE

S.K. Malik : Patterns and Chaos in Fluids

Chairperson : H. C. Khare

11.15-11.30 TEA

11.30-13.00 INVITED TALKS

SESSION - I

Sudhir Ghorpade, C. M. Joshi, P. K. Banerjee

Chairperson : N. K. Thakare

SESSION - II

D. V. Pai, Poornima Pawar, V. K. Gurtu, R. Anand

Chairperson : Hukum Singh

SESSION - III

S. M. Bhave, M. A. Pathan, P. Bhattacharya,

Chairperson : A. K. Agarwal

13.00 - 14.00 LUNCH

14.00 - 15.00 PAPER PRESENTATION :

Session I	:	Chairperson	:	M. A. Pathan
Session II	:	Chairperson	:	S. B. Nimse
Session III	:	Chairperson	:	Manjul Gupta
Session IV	:	Chairperson	:	L. Radhakrishna
Session V	:	Chairperson	:	T. T. Raghunathan

15.00 - 16.45 **SESSION - I**

VEDIC MATHEMATICS SYMPOSIUM SESSION

Co-ordinators : S. L. Singh & Virendra Arora

Inaugural Speaker : Pranav Pandya

Other Speakers : B. S. Rajput, G. P. Pokhariyal, S. R. Pant, P. K. Mukherjee, Vinod Mishra, S. C. Dubey and others.

15.00 - 16.45 **SESSION - II**

SESSION ON GRAPH THEORY

Co-ordinator : T. T. Raghunathan

Speakers : M. M. Shikare, N. S. Bhave

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16.45 - 17.00	TEA		
17.00 - 18.00	GENERAL BODY MEETING		
18.00	VISIT TO SHANTIKUNJ AND DINNER		

DECEMBER 22, 1998

09.30 - 09.45	UNVEILING OF PORTRAIT OF RAMANUJAN		
09.45 - 10.15	R. P. Agarwal	:	Ramanujan's Mathematics In India
10.15 - 11.15	A. K. Agarwal	:	Ninth Ramanujan Memorial Award Lecture "The legacy of Ramanujan".
11.15 - 12.30	TEA		
11.30 - 12.30	**SESSION - I**		
	INVITED TALKS :		
	A. Verma	:	
	R. Y. Denis	:	Ramanujan's Continued Fractions
	S. N. Singh	:	Ramanujan's Mock Theta Function
	Concluding remarks	:	R. P. Agarwal
11.30 - 12.30	**SESSION - II**		
	PAPER PRESENTATION :		
	SESSION VI	:	Chairperson : C. M. Joshi
	SESSION VII	:	Chairperson : Sunder Lal
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	SESSION IX	:	Chairperson : Sudhir Ghorpade
12.30 - 01.00	VALEDICTORY REMARKS BY THE PRESIDENT IMS		
01.00	VISIT TO NIRDHAN NIKETAN AND LUNCH		

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Printed at : KIRAN PRINTING PRESS, Kankhal, Hardwar. Ph.:415975

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ISSN 097-3799

Vol. 12, No. 2, 1998

प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका

JOURNAL OF NATURAL & PHYSICAL SCIENCES

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका
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Journal of Natural & Physical Sciences, Vol. 12 (1998) No.2, 01-13

CONTINUITY OF THE ALGEBRAIC RICCATI EQUATION IN FINITE DIMENSIONAL SPACE

Adam Czornik* and Aleksander Nawrat**

(Received 2.2.98)

ABSTRACT

In this paper the continuity of the solution of the differential and algebraic Riccati equations for continuous time linear quadratic control problem as a function of coefficients is verified. The assumptions for this are stabilizability and observability.

A.M.S. Classification Numbers : 93B25, 93B60

Keywords : Riccati Equation, LQ problem, LQG problem.

INTRODUCTION

The algebraic Riccati equation plays the central role in the computation of the optimal control in the infinite - time deterministic and stochastic linear regulator problem (Theorem 1). The problem of adaptive control of stochastic system combines the problems of identification and control of unknown system [3]. The most desirable solution of a stochastic adaptive control problem is to exhibit a family of estimators that is strongly consistent and to use these estimates to obtain a family of controls that suitably converge to the optimal control for the true system to obtain the self - tuning and self - optimizing properties. The continuity of the algebraic Riccati equation as a function of its

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coefficients if essential for the verification that a family of certainty equivalent controls based on a strongly consistent family of estimates of the unknown parameters provides the self-tuning and self-optimizing properties. In [3] an adaptive control problem for stochastic linear regulator is solved in this case when the unknown parameters appear only in the linear transformations of the state. This solution required the verification of the continuity of the solution of the algebraic Riccati equation as a function of its coefficients. This continuity is verified assuming that the unknown systems are reachable. However, it is desirable to replace this assumption by some more natural system theoretic assumption, such as stabilizability. This is the purpose of this paper.

In the paper [2] the asymptotic behaviour of the algebraic Riccati equation is considered when its coefficients are stabilizale and converge to the unstabilizability coefficients.

PRELIMINARIES

Consider stochastic control system

$$dx_t = (Ax_t + Bu_t)dt + Cdw_t,\ x_0 \equiv x, \tag{1}$$

where $x_t \in R^n, u_t \in R^m, A \in L(R^n, R^n),\ B \in L(R^m, R^n), C \in L(R^n, R^n)$, $w_t, t \geq 0$ is a standard R^n – valued Wiener process and $x_0 \equiv x \in R^n$. The cost functional for finite time horizon is defined as

$$J_T(x_0, u_t) = E\left(\int_0^T \left(\langle Qx_s, x_s\rangle + \langle Ru_s, u_s\rangle\right)ds + \langle Fx_T, x_T\rangle\right), \tag{2}$$

where $Q \in L(R^n, R^n), R \in L(R^m, R^n), F \in L(R^n, R^n)$ are symmetric and Q, F are non-negative, R positive definite and defined as

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$$J(x_0, u_t) = \lim_{T\to\infty} \frac{1}{T} E \int_0^T \left(\langle Qx_s, x_s \rangle + \langle Ru_s, u_s \rangle \right) ds, \tag{3}$$

for infinite time horizon.

The next theorem is well known (see for example, [1]).

Lemma 1. Optimal control for system (1) *with cost functional* (2) *is*

$$\tilde{u}_t = -R^{-1}B'P(T-t)x_t$$

where $P(t)$ *is the solution of the differential Riccati equation*

$$\frac{d}{dt}P(t) = Q + P(t)A + A'P(t) - P(t)BR^{-1}B'P(t) \tag{4}$$

with the initial condition $P(0) = F$, and

$$J_T(u) = \int_0^T tr\left(C'P(t)C\right)dt + \langle P(T)x_0, x_0 \rangle$$

Certain generalisation of classical result of optimal control in the system (1) with cost functional (3) is Theorem 1.

Theorem 1. If a non-negative solution P *of the algebraic Riccati equation*

$$Q + PA + A'P - PBR^{-1}B'P = 0 \tag{5}$$

exists then minimal solution (in the meaning of order of symmetric matrices) $\tilde{P}$ *also exists and control*

$$\tilde{u}_t = -R^{-1}B'\tilde{P}x_t$$

is optimal for the system (1) with cost functional (3). Moreover

$$\tilde{P} = \lim_{t\to\infty} P(t)$$

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where $P(t)$ is the solution of equation (4) with the initial condition $P(0)=0$ and

$$J(x_0,\tilde{u}_t)=tr\left(C'\tilde{P}C\right).$$

Proof : For the proof of existing of the minimal solution consider equation (1) with matrix $C=0$. Lets see those if $P_1(t),P_2(t)$ are two solutions of equation (4) and $P_1(0)\le P_2(0)$ then for all $t\ge 0$ is $P_1(t)\le P_2(t)$. It is because the minimal value of the functional

$$J_t^1(x_0,u_t)=\int_0^t\left(\langle Qx_s,x_s\rangle+\langle Ru_s,u_s\rangle\right)ds+\langle P_1(0)x,x\rangle$$

is not greater then the minimal value of functional

$$J_t^2(x_0,u_t)=\int_0^t\left(\langle Qx_s,x_s\rangle+\langle Ru_s,u_s\rangle\right)ds+\langle P_2(0)x,x\rangle$$

and by Lemma 1 those minimal values equal, respectively $\langle P_1(t)x,x\rangle$, $\langle P_2(t)x,x\rangle$. In particular let $P_1(0)=0$, $P_2(0)=P$. Then $P_2(t)=P$ and because $P_1(t)\le P$ so Lemma 1 implies that the function $P_1(t)$ is non-decreasing (in meaning of order of symmetric matrices). It implicates existence of limit $\lim_{t\to\infty}P_1(t)=\tilde{P}$. Coming to the limit in equation (4) and knowing that $\lim_{t\to\infty}P_1(t)=0$ it is seen that $\tilde{P}$ satisfies equation (5) and is minimal solution.

For the proof of the second part of the Theorem observe that Leema 1 implicates

$$\int_0^T tr\left(C^*P_1(t)C\right)dt+\langle\tilde{P}x,x\rangle=E\int_0^T\left(\langle Q\tilde{x}_t,\tilde{x}_t\rangle+\langle R\tilde{u}_t,\tilde{u}_t\rangle\right)dt+\langle\tilde{P}x_T,x_T\rangle$$

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for all $T \geq 0, x \in R^n$ where $\tilde{x}_t$ is the solution of equations (1) corresponding with control $\tilde{u}_t$. Then we obtain

$$\int_0^T tr\left(C^* P_1(t) C\right) dt + \left\langle \tilde{P}x, x \right\rangle \geq E \int_0^T \left(\left\langle Q\tilde{x}_t, \tilde{x}_t \right\rangle + \left\langle R\tilde{u}_t, \tilde{u}_t \right\rangle \right) dt.$$

Dividing this inequality by T and coming to the limit we have

$$J\left(x, \tilde{u}_t\right) \leq tr\left(C^* \tilde{P} C\right).$$

Because for all $T \geq 0$, $x \in R^n$

$$\int_0^T tr\left(C^* P_1(t) C\right) dt + \left\langle P_1(T)x, x \right\rangle \leq E \int_0^T \left(\left\langle Q\tilde{x}_t, \tilde{x}_t \right\rangle + \left\langle R\tilde{u}_t, \tilde{u}_t \right\rangle \right) dt,$$

so

$$tr\left(C^* \tilde{P} C\right) \leq J\left(x, \tilde{u}_t\right)$$

and finally

$$tr\left(C^* \tilde{P} C\right) = J\left(x, \tilde{u}_t\right).$$

This completes the proof of the Theorem 1.

A sufficient condition for existence of the solution of equation (5) is stabilizability of pair (A, B) ([5] for finite dimensional system and [6] for infinite dimensional). It is not a necessary condition for existence of non-negative definite solution of equation (5), what more by Theorem 1 for the existence of optimal control. Indeed if pair (A, B) is stabilizable and P is the solution of equation (5), then

$$\hat{P} = \begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix}$$

is the solution of equation

$$\hat{Q} + \hat{P}\hat{A} + \hat{A}^* \hat{P} - \hat{P}\hat{B}\hat{R}^{-1}\hat{B}^* \hat{P} = 0$$

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where

$$\hat{A}=\begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix},\ \hat{B}=\begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix},\ \hat{Q}=\begin{bmatrix} Q & 0 \\ 0 & 0 \end{bmatrix},\ \hat{R}=\begin{bmatrix} R & 0 \\ 0 & I \end{bmatrix},$$

and pair $(\hat{A},\hat{B})$ is not stabilizable.

MAIN RESULTS

Initially a suitable continuity of the solution of the differential Riccati equation (4) as a function of its coefficients is verified which is used for the continuity of the solution of the Riccati equation (5).

In further considerations, we consider families of matrix $(A_s)_{s\geq 0}$, $(B_s)_{s\geq 0}$ such, that limit $\lim_{s\to\infty} A_s$ and $\lim_{s\to\infty} B_s$ exist. Consequently exist $s \geq s_0$ and $M>0$ such, that $\|A_s\|<M$ and $\|B_s\|<M$ for $s_0 \geq 0$. For further consideration, and without loosing generality, we can assume, that inequalities $\|A_s\|<M$ and $\|B_s\|<M$ are true for every $s>0$.

Theorem 2 Assume that $A_s \in L(R^n, R^n), B_s \in L(R^m, R^n), s \geq 0$ and

$$\lim_{s\to\infty} A_s = A,\ \lim_{s\to\infty} B_s = B$$

that

$$\lim_{s\to\infty} P_s(t) = P(t)$$

and the converge is almost uniformly, where $P_s(t)$ *and* $P(t)$ *are the solutions of equation*

$$\frac{d}{dt}P_s(t) = Q + P_s(t)A_s + A_s^* P_s(t) - P_s(t)B_s R^{-1} B_s^* P_s(t),\ P_s(0)=0 \qquad (6)$$

and (4) respectively.

To verify this Theorem the following result is important.

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Leema 2. If $A_s \in L(R^n, R^n), s \geq 0$ *and* $\lim_{s\to\infty} A_s = A$, *that*

$$\lim_{s\to\infty} e^{A_s t} = e^{At}$$

and the converge is almost uniformly.

Proof : Fix $T > 0$ and let $t \in [0,T]$. By definition of the exponential of a matrix we have

$$\left\| e^{A_s t} - e^{At} \right\| \leq \sum_{l=0}^{\infty} \frac{T^l}{l!} \left\| A_s^l - A^l \right\|. \tag{7}$$

It easily follows that

$$\left\| A_s^{l+1} - A^{l+1} \right\| = \left\| A_s^l A_s - A_s^l A + A_s^l A - A^l A \right\| \leq \left\| A_s \right\|^l \left\| A_s - A \right\| + \left\| A \right\| \left\| A_s^l - A^l \right\|$$

so that

$$\left\| A_s^{l+1} - A^{l+1} \right\| \leq \left\| A_s - A \right\| \left[\sum_{i=0}^{l} \left\| A_s \right\|^{l-i} \left\| A \right\|^i \right] \leq \left\| A_s - A \right\| (l+1) M^l, \tag{8}$$

where

$$M = \max \left\{ \sup_{s\geq 0} \left\| A_s \right\|, \left\| A \right\| \right\}.$$

From (7) and (8) we obtain

$$\left\| e^{A_s t} - e^{At} \right\| \leq \left\| A_s - A \right\| T e^M.$$

This proves the Lemma.

Proof of Theorem 2: Fix T and let $t \in [0,T]$. Since the limits $\lim_{s\to\infty} A_s$ and $\lim_{s\to\infty} B_s$ exist the from Lemma 1 there is $M_1(T) > 0$ such that

$$\sup_{\substack{s\in[0,\infty) \\ t\in[0,T]}} \left\| e^{A_s t} \right\| \leq M_1(T). \tag{9}$$

It easily follows from (4) and (6) that the function $P_s(t)$ and $P(t)$ satisfies

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$$P_s(t) = \int_0^t e^{A_s^*(t-x)}\left[Q - P_s(x)B_sR^{-1}B_s^*P_s(x)\right]e^{A_s(t-x)}dx. \tag{10}$$

From (9) and (10) we have

$$\|P_s(t)\| \le \int_0^T \left\|e^{A_s(t-x)}\right\|^2 \|Q\|dx \le M_1^2(T)\|Q\|T. \tag{11}$$

Taking difference (6) and (4) we have

$$\|P_s(t) - P(t)\| \le \int_0^t \|A_s^*P_s(x) - A^*P(x)\|dx + \int_0^t \|A_sP_s(x) - AP(x)\|dx$$

$$+ \int_0^t \|P_s(x)B_sR^{-1}B^*{}_sP_s(x) - P(x)BR^{-1}B^*P(x)\|dx$$

$$= 2\int_0^t \|A_sP_s(x) - AP(x)\|dx + \int_0^t \|P_s(x)B_sR^{-1}B_s^*P_s(x) - P(x)BR^{-1}B^*P(x)\|dx$$

$$= 2\int_0^t \|A_sP_s(x) + A_sP(x) - A_sP(x) - AP(x)\|dx$$

$$+ \int_0^t \|P_s(x)B_sR^{-1}B_s^*P_s(x) + P(x)B_sR^{-1}B_s^*P_s(x) - P(x)B_sR^{-1}B_s^*P_s(x) - P(x)BR^{-1}B^*P(x)\|dx$$

$$\le 2\|A_s\|\int_0^t \|P_s(x) - P(x)\|dx + 2\|A_s - A\|\int_0^t \|B_sR^{-1}B_s^*P_s(x)\|$$

$$\|P_s(x) - P(x)\|dx + \int_0^t \|P(x)\|\|B_sR^{-1}B_s^*P_s(x) - BR^{-1}B^*P(x)\|dx$$

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$$\leq 2\|A_s\|\int_0^t \|P_s(x)-P(x)\|dx + 2\|A_s - A\|\int_0^t \|P(x)\|dx + \|R^{-1}\| \sup_{s\in[0,\infty)} \|B_s\|^2 \sup_{\substack{s\in[0,\infty)\\ y\in[0,t]}} \|P_s(y)\| \tag{10}$$

$$\int_0^t \|P_s(x)-P(x)\|dx + \sup_{\substack{s\in[0,\infty)\\ y\in[0,t]}} \|P_s(y)\| \int_0^t \|B_s R^{-1} B_s^* P_s(x) + B_s R^{-1} B_s^* P_s(x) \tag{11}$$

$$-B_s R^{-1} B_s^* P(x) - B_s R^{-1} B^*{}_s P(x) - BR^{-1} B^* P(x)\|dx$$

$$\leq 2 \sup_{s\in[0,\infty)} \|A_s\| \int_0^t \|P_s(x)-P(x)\|dx + 2T\|A_s - A\| \sup_{\substack{s\in[0,\infty)\\ y\in[0,t]}} \|P_s(y)\|$$

$$+\|R^{-1}\| \sup_{s\in[0,\infty)} \|B_s\|^2 \sup_{\substack{s\in[0,\infty)\\ y\in[0,T]}} \|P_s(y)\| \int_0^t \|P_s(x)-P(x)\|dx$$

$$+T\left(\sup_{\substack{s\in[0,\infty)\\ y\in[0,T]}} \|P_s(y)\|\right)^2 \|B_s R^{-1} B^*{}_s - BR^{-1} B^*\|.$$

From (11) we have

$$\sup_{s\in[0,\infty)} \|A_s\| < \infty,\ \sup_{s\in[0,\infty)} \|B_s\|^2 < \infty \text{ and } \sup_{\substack{s\in[0,\infty)\\ y\in[0,t]}} \|P_s(y)\| < \infty$$

so that

$$\|P_s(t) - P(t)\| \leq f(T,s) + g(T)\int_0^t \|P_s(x) - P(x)\|dx, \tag{12}$$

where

$$f(T,s) = 2T\|A_s - A\| \sup_{\substack{s\in[0,\infty)\\ y\in[0,T]}} \|P_s(y)\| + T\left(\sup_{\substack{s\in[0,\infty)\\ y\in[0,T]}} \|P_s(y)\|\right)^2 \|B_s R^{-1} B^*{}_s - BR^{-1} B^*\|,$$

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$$g(T) = 2 \sup_{s\in[0,\infty)} \|A_s\| + \|R^{-1}\| \sup_{s\in[0,\infty)} \|B_s\|^2 \sup_{\substack{s\in[0,\infty)\\ y\in[0,t]}} \|P_s(y)\|.$$

Apply Gronwall's lemma to (12) to obtain

$$\sup_{0\le t\le T} \|P_s(t) - P(t)\| \le f(T,s)e^{Tg(T)}.$$

It follows that

$$\lim_{s\to\infty} f(T,s) = 0,$$

so the proof is complete.

Theorem 3. If $A_s \in L(R^n, R^n), B_s \in L(R^m, R^n), s \ge 0,$

$$\lim_{s\to\infty} A_s = A, \ \lim_{s\to\infty} B_s = B,$$

and $A - BR^{-1}B^*P$ *is stable where* P *is the minmal solution of (5), then exists* $s_0 \ge 0$ *that for* $s \ge s_0$ *equation*

$$Q + PA_s + A_s^*P - PB_sR^{-1}B_s^*P = 0 \tag{13}$$

has solutin and $\lim_{s\to\infty} P_s = P$ *where* P_s *is the minimal solution of* (13).

To verify this Theorem, next three lemmas are need.

Lemma 3 *Let* P *be the solution (5) and* $P(t)$ be the solution *(4) with initial condition* $P(0)$, *where* $P(0) \ge P$. *Then, for each* $t \ge 0$,

$$\|P(t) - P\| \le \left\|(P(0) - P)^{1/2}\right\|^2 \left\|e^{(A-BR^{-1}B^*P)t}\right\|^2. \tag{14}$$

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Proof. It follows that $S(t) = P(t) - P$ is a solution of equation :

$$\frac{d}{dt}S(t) = \left(A - BR^{-1}B^{*}P\right)^{*} S(t) + S(t)\left(A - BR^{-1}B^{*}S\right) - S(t)\,BR^{-1}B^{*}S(t)$$

Thus

$$S(t) = \left(e^{(A-BR^{-1}B^{*}P)t}\right)^{*} Y(0)\left(e^{(A-BR^{-1}B^{*}P)t}\right) - \int_0^t \left(e^{(A-BR^{-1}B^{*}P)(t-x)}\right)^{*}$$

$$\times s(x)BR^{-1}B^{*}S(x)\left(e^{(A-BR^{-1}B^{*}P)(t-x)}\right)dx$$

so, for $x \in R^n$

$$\langle S(t)x, x\rangle \le \left\langle S(0)e^{(A-BR^{-1}B^{*}P)t}x, e^{(A-BR^{-1}B^{*}P)t}x\right\rangle. \tag{15}$$

The nonnegativity of $S(0)$ implies the nonnegativityof $S(t)$ so that (15) implies (14).

Lemma 4 : To each $A \in L\left(R^n, R^n\right)$ *and to each open set* σ_0 *that includes* $\sigma(A)$ *there corresponds a positive number* ε *such that if* $\left\|A - \widetilde{A}\right\| < \varepsilon$ *trhen* $\sigma(\widetilde{A}) \subset \sigma_0$.

Proof of this Theorem we can find for example in [4].

Lemma 5 : If $A_s \in L\left(R^n, R^n\right)$, $B_s \in L\left(R^m, R^n\right), s \ge 0,$

$$\lim_{s\to\infty} A_s = A, \quad \lim_{s\to\infty} B_s = B,$$

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(A,B) is stabilizable and $A-BR^{-1}B^{*}P$ is stable, where P is the minimal solution of equation (5) then there are real numbers $s_0>0$, M and $\delta>0$ such that for $t\geq 0$ and $s\geq s_0$,

$$\left\|e^{(A_s-B_sR^{-1}B_s^{*}P)t}\right\|\leq Me^{-\delta t}. \tag{16}$$

Proof This Theorem is consequence of Lemma 2, Lemma 4 and this fact that for each matrix $A\in L(R^n,R^n)$ next three conditions are equivalent :

1. Matrix A is stable
2. For each $\lambda\in\sigma(A)$ is $\mathrm{Re}(\lambda)<0$
3. There are real numbers M and $\delta>0$ such that for $t\geq 0$ is $\left\|e^{At}\right\|\leq Me^{-\delta t}$.

Proof of Theorem 3 : There is $s_0>0$ such that for $s\geq s_0$ paid (A_s,B_s) is stabilizable, so for $s\geq s_0$ equation (13) has a solution [5]. Let P_s be the minimal solution. It follows from Lemmas 3 and 5 that there exist numbers $\tilde{M}$ and $\delta>0$ such that

$$\left\|P_s(t)-P_s\right\|\leq \tilde{M}e^{-\delta t}$$

for $s\geq s_0$ and $t\geq 0$. Given $\varepsilon>0$ there is a $T(\varepsilon)>0$ by (14) such that

$$\left\|P_s(T(\varepsilon))-P_s\right\|<\frac{\varepsilon}{3}\text{and}\left\|P(T(\varepsilon))-P\right\|<\frac{\varepsilon}{3}$$

for $s\geq s_0$. Since $\lim\limits_{s\to\infty}\left\|P_s(T(\varepsilon))-P(T(\varepsilon))\right\|=0$ by Theorem 2. Hence it follows that there is a $s(\varepsilon,T(\varepsilon),s_0)>0$ such that if $s>s(\varepsilon,T(\varepsilon),s_0)$

$$\left\|P_s(T(\varepsilon))-P(T(\varepsilon))\right\|<\frac{\varepsilon}{3}.$$

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Finally,

$$\left\|\widetilde{P}_s - \widetilde{P}\right\| \le \left\|\widetilde{P}_s - P_s\big(T(\varepsilon)\big)\right\| + \left\|P_s\big(T(\varepsilon)\big) - P\big(T(\varepsilon)\big)\right\| + \left\|P\big(T(\varepsilon)\big) - P\right\| < \varepsilon,$$

which completes the proof.

REFERENCES

[1] M. Arato, *Linear stochastic systems with constant cofficients a statistical approcah*, Springer-Verlag, Berlin Heidelberg New York, 1982.

[2] **A. Czornik and W. Sragovich, *Об асимптотическом поведении алгебраического уравнения Риккати для непрерывного времени*, Автоматика и Телемеханика 8(1995), 90 - 92**

[3] T.E. Duncan and B. Pasik - Duncan, *Adaptive control of continuous time lineat systems*, Mathematical Control Signal System, 3 (1990), 45-60.

[4] P.R. Halmos, *A Hilbert space problem book, D. Van Nostrand Company, inc., 1967*

[5] W.M. Wonham, *On pole assignment in multi-input controllable linear systems*, IEEE Trans. Automat. Control, AC-12(1987), 660-665.

[6] J. Zabczyk, *Remarks on the algebraic Riccati equation in Hilbert Space*, Applied Mathematics and optimization, 2(1976), 251-258.

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका, खण्ड 12 (1998) अंक 2, 14
Journal of Natural & Physical Sciences, Vol. 12 (1998) No.2, 14

ON GENERAL HYBRID CONTRACTIONS : CORRIGENDUM

S. L. Singh* and S. N. Mishra**

The authors [1] have presented several coincidence and fixed point theorems for single-valued and multivalued maps on an arbitrary set with values in a metric space (x,d). However, Example 4 [1] needs minor corrections. Indeed, in Example 4([1], page 252), "$Tx = x/3$ for all $x \in X$" should be corrected to "$Tx = (x+2)/27, x \le 0, Tx = (x-2)/27, x > 0$"; "Therefore, conditions [9, (4.2) and (4.3)]" to "Therefore, conditions [9, (4.2) with the interchange of the role of T and f and (4.3)]"; and "versions of [9, Corollary 3.4]" to "versions of [9, Corollary 4.3]".

REFERENCE

1. S.L. Singh and S.N. Mishra : On general contractions, J. Austral. Math. Soc. (Series A) 66 (1999), 244-254.

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Journal of Natural & Physical Sciences, Vol. 12 (1998) No.2, 15-22

ON ADAPTIVE CONTROL FOR LINEAR STOCHASTIC SYSTEM

Adam Czornik* and Aleksander Nawrat**

ABSTRACT

The one dimensional, continuous time stochastic given by the maximum likelihood method are used to define the feedback gain, and control system with complete observation is considered with quadratic cost functional. The parameter estimates the adaptive control is taken to be a linear-state feedback accordance with stopping times defined on the trajectory of the system. It is prove that the parameter estimates are strongly consistent and the cost functional reaches its minimum, i.e the adaptive control is optimal.

INTRODUCTION

Many physical systems are naturally modeled by linear, continuous differential equations. Since some parameters of the system are often unknown, it is important to study the adaptive control of linear, continuous differential equations. The problem of the adaptive control of linear one-dimensional continuous differential equation is investigated in this paper and a solution is obtainded. A solution of the adaptive control problem means, that the average costs for the control problems using the parameter estimates converge almost surely to the optimal average cost.

There exist many papers about the parameter estimates and adaptive control in linear continuous differential equations, see [1]-[3], also [5]-[7] and

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[11]. The assumptions, which are done in those papers are different from this, which we made. And for example in papers [5]-[7] is assumpted knowledge about the connected closure of the range, into which inside belong estimates' parameters. Moreover in papers [5] and [7] are ordered some specified asymptotic behaviour of determinant. In ours paper succeeded to omit these assumptions in one-dimensional case.

On the other hand, taken restriction on the control, in papers [1]-[3], [8] and [11], make impossible to use these result to considered cost functional by us.

In this paper we consider the one-dimensional stochastic differential equation

$$dx_t = (ax_t + bu_t)dt + dW_t, \quad x_0 = x, \tag{1}$$

where $x_0, x_t, u_t \in R$, the process u_t $t \geq 0$ is nonanticipative and W_t, $t \geq 0$ is a standard one dimensional Brownian motion. It is assumed, that the known parameter $a \in R$, and the unknown parameter $b \in R, b \neq 0$.

At any time $t \geq 0$ based on the measurements $x_s, 0 \leq s \leq t$ we hope to estimate the unknown parameter b and to design the adaptive control with the purpose of minimizing the cost functional :

$$C = \lim_{t \to \infty} \frac{1}{t} C_t, \tag{2}$$

where

$$C_t = \int_0^t (qx_s^2 + ru_s^2)\, ds, \tag{3}$$

and $q \geq 0, r > 0.$

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The contribution of this paper is that an adaptive control is given such, that the cost functional (3) is minimized (Theorem 3) and the parameter are strongly consistent (Theorem 1).

2. Parameter estimates and adaptive control law.

In this section we first define the maximum likelihood estimate $\hat{b}(t)$ for b and then define the adaptive control u_t on the basis of observations $x_s, 0 \le s \le t$ and the estimate $\hat{b}(t)$ obtained as well.

The likelihood function is obtained from the mutual absolute continuity of the probability measure of $x_t, t \le 0$ and the Wiener measure for $W(t), t \le 0$, (see [7]) as

$$L(t,b) = \exp\left[\int_0^t (ax_s + bu_s)dx_s - \frac{1}{2}\int_0^t (ax_s + bu_s)^2 ds\right]. \tag{4}$$

The maximum likelihood estimate $\hat{b}(t)$ of b is obtained by maximizing $L(t,b)$. To maximizing $L(t,b)$, it is necessary (and in fact sufficient), that

$$\frac{din\ L(t,b)}{db} = 0. \tag{5}$$

Solving (5), we obtain

$$\hat{b}(t) = \frac{\int_0^t u_s dx_s - a\int_0^t x_s u_s ds}{\int_0^t u_s^2 ds} \tag{6}$$

Now use $\quad dx_t = (ax_t + bu_t)\,dt + dW_t, \quad x_0 = x$

and rewrite (6), we have

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$$\hat{b}(t) - b = \frac{\int_0^t u_s dW_s}{\int_0^t u_s^2 ds} \tag{7}$$

We now proceed to define the adaptive control. It is well known (see, for example, [4]), that the feedback control $u_t = kx_t$, where $k = -\frac{a + \sqrt{a^2 + \frac{qb^2}{r}}}{b}$, makes the cost functional (2) minimal and equal to $C = \frac{ar + \sqrt{a^2r^2 + b^2qr}}{b^2}$. However, this optimal control can not be used here, since b is unknown. It is natural to replace b by their estimates $\hat{b}(t)$. If $\hat{b}(t) \neq 0$, we define k_t, as follows :

$$k_t = -\frac{a + \sqrt{a^2 + \frac{q\left(\hat{b}(t)\right)^2}{r}}}{\hat{b}(t)}, \tag{8}$$

Take any $k_0 \neq 0$, as the initial value for adaptive feedback gain. The time axis is partitioned by stopping times $\tau_n, n = 0,1,2,...$ and the adaptive control is defined as follows :

$$u_t = \begin{cases} k^{\tau_n} x_t & \text{if } t \text{ belongs to some} [\tau_n, \tau_{n+1}) \text{ and } \hat{b}(\tau_n) \neq 0 \\ k_{\tau_{n-1}} x_t & \text{if } t \text{ belongs to some} [\tau_n, \tau_{n+1}) \text{ and } \hat{b}(\tau_n) = 0. \end{cases} \tag{10}$$

The stopping times are given in this forms :

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$$\tau^0 = 0 \tag{11}$$

$$\tau_n = \begin{cases} \inf\left\{t : \int_0^t u_s^2 ds \geq n\right\} \\ \infty \quad \text{if the above set is empty.} \end{cases}$$

3. Strong consistency of parameter estimates and optimality of adaptive control.

We first prove a Lemma

Lemma

For any n, τ_n is finite a.s.

Proof :

Assume that there is a set $\wedge \subset \Omega$ with $P(\wedge) > 0$ and the number $n_0 \in N$, such that, for $\omega \in A$

$$\tau_{n_0}(\omega) = \infty$$

that means

$$\int_0^\infty u_x^2(\omega) ds \leq n_0$$

then by definition of the adaptive control and τ_n, we have

$$\int_0^\infty x_s^2(\omega) ds < \infty \tag{12}$$

because the feedback gain takes only finite numbers of value : $k_{t_0}, k_{t_1}, \ldots\ldots k_{t_{n0}}$ and they are all nonzero.

A contradiction form (12) will be established . Itô's formula implies that :

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$$x^2_{t\wedge t_{n0}} - x_0^2 = 2\int_0^{t\wedge t_{n0}} (ax_s + bu_s)x_s ds + 2\int_0^{t\wedge t_{n0}} x_s dW_s + t\wedge\tau_{n0} \tag{13}$$

The first integral in (13) can be bounded using the Schwarz inequality and the boundedness on feedback gain and the second integral is a square integrable martingale by the definition of stopping times. Letting $t\to\infty$, it follows, that for $\omega\in\wedge$

$$\lim_{t\to\infty} x_t^2(\omega) = \infty$$

However this contradicts the assumption (12)

Theorem 1

Under the adaptive control defined by (10) the family of maximum likelihood estimates $\hat{b}(t)$ is strongly consistent, that means :

$$P_b\left(\lim_{t\to\infty}\hat{b}(t) = b\right) = 1.$$

Proof :

To verify this Theorem the following result [10] is important. If (W_t, F_t), $t\geq 0$ is a Wiener process on the probability space (Ω, F, P) and the process (f_t) is F_t- adapted for $t\geq 0$, such that

$$\int_0^t f_s^2 ds < \infty \ a.s \ for \ 0\leq t<\infty$$

and

$$\int_0^t f_s^2 ds = \infty \ a.s,$$

then the process (z_s, G_s) for $s\geq 0$, where $z_s = \int_0^{\tau_s} f_t dW_t$, $G_s = F_{t_s}$ and

$$\tau_s = \inf\left\{t : \int_0^t f_u^2 du > s\right\},$$

is a Wiener process and

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$$\lim_{t\to\infty}\frac{\int_0^t f_s dW_s}{\int_0^t f_s^2 ds}=0 \ a.s.$$

The proof follows from this result, lemma and (7)
In paper [7] was shown the following theorem :

Theorem 2

If $\lim_{t\to\infty} k_t = -\dfrac{a+\sqrt{a^2+\dfrac{qb^2}{r}}}{b}$ *then control* $u_t = k_t x_t$ *is optimal for system (1) with cost functional (2) i.e.* $\lim_{t\to\infty}\dfrac{C_t}{t}=p$.

We now formulate our main result.

Theorem3

The control given by (10) is optimal for system (1) with cost functional (2)

Proof: Theorem 1 has established the convergence $\hat{b}(t)$ to b, as $t\to\infty$, so

$$\lim_{t\to\infty} k_t = k, \tag{14}$$

where $k=-\dfrac{a+\sqrt{a^2+\dfrac{qb^2}{r}}}{b}$ and k_t is given by

$$k_t=\begin{cases} k^{\tau_n} & \text{if } t \text{ belongs to} [\tau_n,\tau_{n+1}) \text{ and } \hat{b}(\tau_n)\neq 0 \\ k_{\tau_{n-1}} & \text{if } t \text{ belongs to some} [\tau_n,\tau_{n+1}) \text{ and } \hat{b}(\tau_n)=0 \end{cases}$$

Now the proof follows from Theorem 2.

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ACKNOWLEDGEMENT

The authors, thanks Professor W. Sragovich and the Referees, for their suggestions, that have improved the clarity and the readability of this paper.

REFERENCES

[1] P.E. *Caines, Recent results in continuous-time stochastic adaptive control: Nonexplosion, consistency and atability*, in Proc. ECC, Grenoble, 1991.

[2] H.F. Chan and L. Guo *Continuous-time stochastic adaptive tracking: robustness and asymptotic properties*, SIAM J. Control Optim. 28(1990), 513-527.

[3] H.F. Chan and J.B. Moore, *Convergence rate of continuous-time ELS Parameter estimation*, IEEE Transaction on Automatic Control, vol. AC-32(1987), no.3, 267-269.

[4] M.H.A Davis , *Linear Estimation and Stochastic Control,* Chapman and Hall, London, 1977.

[5] T.E. Duncan, P. Mendl and B. Pasik- Duncan, *On the consistency of least squares idendtification procedure*, Kybernetiks 24(1988), 340-346.

[6] T.E. Duncan, P. Mendl and B. Pasik-Duncan, *Control theory methods for consistency in some least squares identification problems*, IEEE Transactions on Automatic Control, 38(1993), 1289-1992.

[7] T.E. Duncan and B, Pasik- Duncan, *Adaptive control of continuous time linear systems*, Math. Control Signals Systems, 3(1990), 45-60.

[8] I. Guo, *A note on continuous-time* ELS, System & Control Letters, 22(1994), 111-121.

[9] M. Gevers, G. C. Goodwin and V. Wartz, *Continuous-time stochastic adaptive control*, SIAM J. Contr. Optimiz.,(29)1991, 264-282.

[10] R.S. Liptser and A.N. Shiryayev, *Statistics of random processes*, New York: Springer 1977.

[11] J.B. Moore, Convergence of continous time stochastic ELS parameter estimation, Stochastic Proc. Their Appl., 27(1988), 195-215.

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका, खण्ड 12 (1998) अंक 2, 23-30
Journal of Natural & Physical Sciences, Vol. 12 (1998) No.2, 23-30

OPTICAL SIGNAL CORRELATION AND JOINT FOURIER TRANSFORM

Bharti Bhardwaj* & P.P. Pathak*

(Received 6.9.98)

ABSTRACT

Correspondence between two optical signal is an important part of optical data processing for pattern recognition and forensic sciences applications. Correlation operation is very useful in these kind of circumstances. Its realisation in pracice is possible by utilizing the property of a lens to Fourier Transform a transmittance function present in its first focal plane. A matched filter helps in correlation operation but its designing is not easy. Alternative methods have been developed for implementing correlation. A study of these methods shows that Joint Fourier Transform is a very effective and easy to implement teachnique of achieving optical correlation for the purpose of achieving optical correlation.

Keywords : Aperiodic function, correlation, Joint Fourier Transform, cross-correlation, auto-correlation, power speedring, matched filter, optical correlation, Joint transform correlation.

INTRODUCTION

Correlation operation is the comparison of two signals to determine the degree of correspondence between them [1]. The correlation of two aperiodic functions $f_1(x)$ and $f_2(x)$ in spatial coordinate x is the function

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$$\phi_{12}(\tau) = \int_{-\infty}^{\infty} f_1(x)\, f_2\,(x+\tau)\, dx$$

is called cross-correlation, being the displacement between the functions. Frequencies common to both functions are present in it.

When $f_1(x)$ and $f_2(x)$ are identical

$$\phi_{11}(\tau) = \int_{-\infty}^{\infty} f_1(x)\, f_1(x+\tau)\, dx$$

is called auto-correlation. Frequency components present in $f_1(x)$ are present in it.

In order to obtain the displacement a function is slided over the other, e.g. two identical photographs. As one negative slides over its duplicate the amount of light transmitted varies. Maximum amount of light is transmitted when images of the two negatives coincide i.e. maximum correlation is present.

Spectrum of auto-correlation function of a periodic signal is proportional to the square of the magnitude of the signal. It contains the same information as the power spectrum. Phase information is neither contained in power spectrum nor in auto correlation function.

FOURIER TRANSFORM OF A SPATIAL FUNCTION

Fourier Transform $F(u)$ of a spatial $f(x)$ represents the amplitude and phase of various spatial frequency components of the function $f(x)$.

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

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u is called "spatial frequency" and has units of reciprocal length.

POWER SPECTRUM

In optics the intensity of power of light is proportional to the square of the light amplitude. So the power spectrum can be obtained by squaring the absolute values of the Fourier Transform.

$$I(u) = |F(u)|^2$$
$$= F(u)\, F^*(u)$$

Thus power spectrum is the magnitude of Fourier Transform squared and can be obtained by multiplying the Fourier Transform by its complex conjugate.

MATCHED FILTER

If an input waveform of known amplitude and phase spectra is fed into a system that has a known amplitude, frequency and phase response, the output amplitude spectrum is found by multiplying the input amplitude spectrum by the system amplitude frequency response. The phase of the output is the summation of input phase spectrum and the system phase response.

If (i) system amplitude frequency response equals the input amplitude spectra, i.e.

$$R(u) = A(u)$$

and (ii) phase frequency response of the system be equal and opposite to the input phase spectrum, i.e.,

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$$\angle \psi(u) = - \angle \phi(u)$$

Then the amplitude spectrum of output is equal to the input power spectrum and output phase spectrum is zero.

$$A(u)\, R(u) = |A(u)|^2$$

$$\angle [\psi(u) + \phi(u)] = 0$$

such a system is called "Matched Filter".

OPTICAL CORRELATOR

An optical correlator requires

(i) An arrangement to Fourier Transform a given transmission function.

(ii) A matched filter to convert Fourier transformed signal intgo power spectrum.

The development of a matched filter involves holographic techniques and is an involved process. It is further difficult to generate rapidly changing matched filters specially when results are required in quick succession.

An arrangement to produce optical correlation is shown in the figure (ii). A collimated coherent beam of light having amplitude A_0 is incident on the optical transparancy, i.e. transmission function $f_1(x)$ which transmits A_0 $f1(x)$ light. This optical signal being in the front focal plane of lens 2 is Fourier transformed into back focal plane of this lens. (A lens Fourier transforms a function placed in its front focal plane).

Lens 3 further Fourier transforms it and produces transform of transform

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into its back focal plane called reference plane. This is nothing but inverted image of $A_0 f_1(x)$. A second transmission function $f_2(x)$ in form of optical transparancy is introduced at the reference plane with proper orientation. Amplitude of light transmitted from reference is the product of the incident light amplitude and transmission function in the reference plane, i.e., $A_0 f_1(x)$ $f_2(x)$. This also being the front focal plane of lens 4, Fourier transform of the product is performed this lens.

Since correlation involves displacement of a funcion and displacement in reference is undesirable it is done in $f_1(x)$ and not in $f_2(x)$. For this a function is slided over the other. Fourier transform so obtained has exponential terms.

$$\int f_2(x)\, f_1(x+\tau)\, e^{-j2\pi ux} dx$$

Since correlation does not have exponential term it can be set equal to unity by taking $u = 0$ i.e. zero order term. So the zero order spatial spectrum term in output plane is thus correlation function. This can be monitored by a photocell or recorded on a photographic film.

If $f_1(x)$ and $f_2(x)$ are different it is cross-correlation and if they are identical it is auto-correlation.

JOINT TRANSFORM CORRELATOR

Schedule representation of joint transform corrlator is shown in the figure (iii) Input functions input plane side by side as shown. The amplitude transmittance of input plane in one dimension is given by [2]

$$u_1(x_1) = g\,(x_1 - b) + h\,(x_1 + b)$$

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when $2b$ is centre to centre spacing of the two functions and the physical limitations of them is 'b'. Lens L_1 forms Fourier transform of u_1 at transform plane P_2. The transmittance of P_2 assuming it to be proportional to the light intensity is given by

$$t(x_2, y_2) = |u_2(x_2, y_2)|^2$$

$$= |G(u, v)\, e^{-j2\pi ub} + H(u,v) e^{+j2\pi ub}|^2$$

$$= |G|^2 |H|^2 + GH^* e^{-j4\pi ub} + G^* H e^{j4\pi ub}$$

when this function $t\,(x_2, y_2)$ is illuminated as shown in the figure its Fourier transform is formed by lens L_2 at plane P_3. Accordingly light amplitude distribution at plane P_3 will be

$$u(x_3, y_3) = g^* g - h * h - g * h * d\,(x_3 - 2b) + h * g * \delta\,(x_3 + 2b).$$

It is thus clear that u (x_3, y_3) contains the correlation spots centred at $x_3 = 2b$. For simplicity focal lengths of L_1 and L_2 are assumed to be equal.

If input functions g and h, are identical Joint Transform shows well defined peaks. These are called correlation peaks. At times these peaks are not well defined and are immersed in noise, and it becomes difficult to analyse them. So for the better identification of optical patterns there is every need for modification or enhancement in these peaks of joint Transform power spectrum [3].

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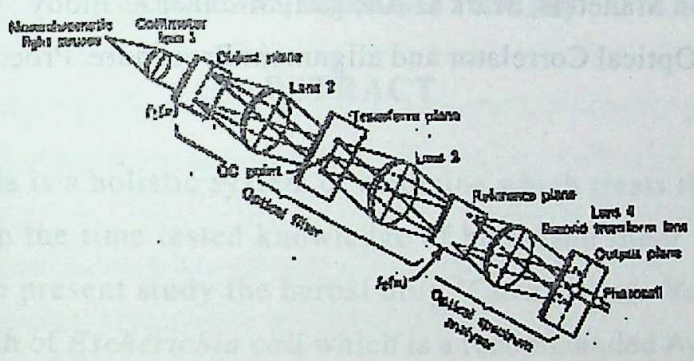

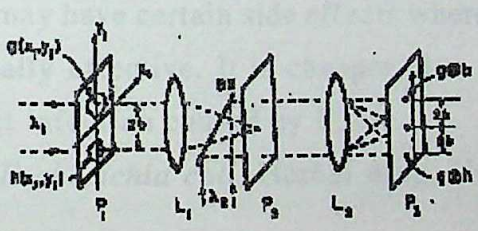

CONCLUSION

Various methods including modification of Joint Power Specturm utilizing non-linear techniques have been used for the enhancement of correlation. The several noise removal techniques have also been used to optimize the results. In order to implement the optical correlation techniques in forensic sciences work should be undertaken to enhance the peaks and optimize the noise removal techniques for better and better identification of different patterns in these applications.

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REFERENCES

1. Arnold Roy Shulman : Optical Data Processing, John Wiley & Sons Inc. New York, 1970.
2. D. Casasent and H. J. Cauffield : Optical Data Processing Spiger Verlag Berlin Heilberg, New York 1978.
3. Damon Manettas, Mark L' Antigua, Michael A. Fiddy : Commercialisation of an Optical Correlator and alignment Procedure Proc. SPIE 1822, (1992) 33-42.

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका, खण्ड 12 (1998) अंक 2, 31-34
Journal of Natural & Physical Sciences, Vol. 12 (1998) No.2, 31-34

ANTIMICROBIAL EFFECT OF HERBAL DRUG "CHANDANADI-VATI" IN E.COLI. INFECTION

P. Singh* and Manisha*

(Received 14.9.98 Revised 14.10.98)

ABSTRACT

Ayurveda is a holistic system of medicine which treats the patients as a whole through the time tested knowledge of herbs and other natural products. During the present study the herbal drug "Chanadanadi-Vati" is used to check the growth of *Escherichia coli* which is a recommended Ayurvedic drug against diarrohea and for urinary tract infection (UTI). The sensitivity of the antimicrobial activity of "Chandanadi-Vati" and effective antibiotics are quiet comparable. The antibiotics may have certain side effects whereas herbal drug is harmless while being equally effective. It is cheaper also. Its use is thus recommended in urinary tract infection caused by E.coli.

Key words- Antimicrobial, *Escherichia coli*, Herbal drug, Urinary tract infection.

Ayurvedic medicines mostly obtained from drug plant may be considered as compound drug, as they are mixture of several chemical substances. Their principal active ingredients can be isolated and evaluated against human pathogens to enhance their tharapeutic spectrum. All these active principals are present in storage organs of plants collectively known as 'Panchang', Two of the vedas namely 'Atharaveda' and 'Rigveda' mentioned diseases and their treatment with the help of herbs as well as by making use of few metals and minerals. Many researchers have performed experiments on

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the effect of various herbal extracts on different microbial species. Behl [2] separated out essential compound from *Tinospora cardifolia* and studied its antibactrial effect against *Escherichea coli* Yadav[13], Rao et al. [11], Bhatt et al. [1], Varadraj and shankaranarayana [12], Nand[9], Kaushik[6], Gundidza and Sibanda[5], Prasad and Rao [10] Gupumahadevana et al.[4], Kaushik[7,8] and Gehlot [3] studied the antimicrobial activity of various plant extracts and herbal drugs against *E.coli, Staphylococcus aureus*, *Salmonella typhii*, *Bacillius species* and various other Gram positive and negtive bacteria. Keeping this in mind the present study was conducted to evaluate the antimicrobial effect of herbal drug "ChandanadiVati" which is most commonly used for the treatment of uninary tract infaction and diarrhhoea.

Urinary tract infaction (UTI) is a commin disorder at all ages and in both sexes. The bacteria which is most commonly involved in it is *Escherichia coli* which is responsible for about 80% of UTI. It has many uropathogenic strains, all of which are faecally derived. The present clinical study was done on urine sample of the patients of UTI. The test organism '*E.coli*' was isolated & purified on the nutrient agar with large white, thick, greyish smooth translucent colonies while on Mac Conkey agar medium, the colonies are bright & pink because of fructose fermentation. The *E.coli* is aerobic or facultative anaerobic microorganism growing in the temperature range of 28 to 37^{0}C.

The herbal drug "Chandanadi Vati" was procured from 'Gurukul Kangri Pharmacy'. The main constituents of "Chandanadi Vati" are Mulhatti, Sheetal Cheeni, Kalmishora, Gound babbol and Chandan oil, Rumi-Mastagi, Kabab Cheeni oil.

During the study, the antimicrobial sensitivity of antibiotics as well as of Chandanadi Vati were tested against *E.coli*. As a guide to the sensitivity of specific microorganism to an antibiotic the Minimum inhibitory Concentration (MIC) is used which is defined as the lowest concentration of an antibiotic required to inhibit the growth of a given strain of micro-organism under the controlled condition. The sensitivity of herbal drug "Chandanadi Vati" was observed and compared with the standard antibiotic disc with the

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aim to evaluate the efficacy of both the drug against *E.coli.*

A significant response was exhibited by test-organism to Ayurvedic drug as well as against certain antibiotics. A number of antibiotics such as Ampicillin, Tetracyline, Ofloxacin, Pefloxacin, Amikacin, Ceftizoxime, Ciprofloxacin, Piperacillin, Cefotaxime, Gentamycin and Norfloxacin were examined. Antibiotics 'Norfloxacin' and 'Gentamycin' showed maximum inhibition zone (20mm.) Whereas herbal drug "Chandanadi Vati" proved quite, effective against *E.coli* showed inhibition zone of 18 mm. Thus its efficacy is comparable to that of specific antibiotics. The antimicrobial activity of different antibiotics was highly selective as recorded by earlier workers. Since most of the antibiotics are semisynthetic in origin, some of the antibiotics were prepared from many synthetic chemicals can have certain side effects on the body. The herbal drug "Chandanadi Vati" on the other hand being derived from natural products has no side effects and can also be used against the infection of *E.coli*. The present findings are favourably compared with the result of previous workers [4,9,12,13] Gopumahadevan et.al.,Nande, Varadaraj & Yadav.

In view of above findings it is recommended that the "Chandanadi Vati" (herbal drug) would be safer to human beings because it does not show any side effect and will be as effective as the new drug therapy based on allopathic treatments. It is also recommended that further studies can also be used for the control of other micro-organism so as to establish the new drug therepy in controlling some common pathogens causing ill effect in human beings. Further it would be economically cheap and safe in practice without any side effect.

REFERENCES

1. R.M. Bhatt and S. Karo : Clinical and experimental Studies of 'Panchanalkal' and 'Shatavari' of bourn wound species - bacterial and fungal. J. Natl. Integ; Med. Assoc. 26 (5) (1984); 131.

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2. J. Behl : Inhibition of *E.coli* and Staphylococcus Spp. by the essential compounds of *Tinospora cordifoli*, Indian perfum. 26 (3) (1954) 48-50.

3. D. Gehlot and A. Bohra : Antimicrobial activity of various plant part extracts of *Aerva persica* Adv. Plant Sci 11(1) (1998); 109-111.

4. S. Gopumahadevan, B.L. Chauhan and C. Raju : Effect of herbal preparation 'Septilin' against *E.coli* and *S. aureus*. Ind.J. of experimental Biology. 32 (8) (1994) 553.

5. M. Gundidza and M. Siband : Antimicrobial activities of *Zizyphus abyssinica* and *Berchemia* discolor C. African J. Med. 37 (3) (1992); 80.

6. P. Kaushik and N. Kishore : Antibacterial potential of *Pholidota articulata*- A study in vitro J. Orchid Soc. India, 5 (1,2) (1991); 93-96.

7. P. Kaushik and N. Kishore : Antibacterial activity of *Dendrobium amoenum* Wall. Ex Lindi. - A study in *vitro* J. Orchid Soc. India 9 (1-2) (1995); 33-35

8. P. Kaushik and N. Kishore : In vitro antibacterial activity of *Clisostoma micranthum* K and P Ad. Plant Sci. (2) : Supplement 81-84, (1997).

9. Nand : Inhibition of *E.coli* by using extracts of some medicinal plants. Curr. Sci. 55 (2), (1989), 141.

10. R.J. Prasad and A. Rao : Antimicrobial studies on the seed oil of *Brassica Juncea*, fitoterapia 64 (4) (1994), 373.

11. D.V. Rao, B. Bhagirathi and K.R.G. Rao : Incedence of drug resistant coliform in some ready to serve foods. Arogya J. Health Science., 106 (1979), 112.

12. P. Vardaraj and Shankaranarayana : Antibacterial activity of sandal bark tannins against *Staphylococcus* and *E.coli*. Microbio. cell. Bio.Lab. Indian Inst. Sci; Bangalore, Van Vigyan, 24 (3) (1986), 121.

13. J.N.S. Yadav : Antibiotic sensitivity of *E.coli* strains isolated from Poultry Septicaemic. Indian J. Exp. Bio. 8 (1970), 28.

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका, खण्ड 12 (1998) अंक 2, 35-46
Journal of Natural & Physical Sciences, Vol. 12 (1998) No.2, 35-46

STUDIES ON THE ROLE OF SOLVENT IN THE PERIODATE OXIDATION OF SOME ANILINES

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(Received 3.12.98)

ABSTRACT

The results of Kinetic studies related to the effect of solvent and temperature for the periodate oxidation of fifteen anilines have been presented and discussed. On the basis of the studies and earlier work, a mechanism for this reaction series has been proposed. Some of the results have been found to differ from earlier reported trends.

Kew words : Role of solvent, periodate oxidation, anilines.

INTRODUCTION

The oxidation of aromatic amines with different oxidizing agents provides interesting possibilities, such as a number of complex products are formed, out of which some are important from physiological and industrial point of view.

The studies on oxidation of aromatic amines by periodate ion as reported in literature are very few [1-13]. The kinetic studies made on these non-malapradian oxidations are even fewer. The products formed are pH dependent [6, 11-13]. Earlier reports are contradictory as far as the mechanism of these processes is concerned. Pavolva et.al [7] and Tanabe [11-13] suggested a free radical mechanism for oxidation of few anilines while others [1-10] proposed that the reactions are of ion-dipolar type. Generally stoichiometry of 2 mol oxidant: 1 mol substrate has been

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reported while a first order dependance of rate on each of the reactants has been established in many cases [1-10]. Earlier work suggests for more studies from mechanistic point of view. The present paper is an attempt to investigate and to interpret the role of solvent in these reactions. The results of periodate oxidation of fifteen anilines have been presented and discussed.

MATERIALS AND METHODS

Different anilines and sodium meta periodate used were of Loba G.R.grade. Different anilines and sodium metaperiodate were used after redistillation/recrystallization. The reactions being sensitive to pH were studied at pH 6.5 maintained by using Thiel, Schultz and Coch buffer [14]. The reactions being quite fast at ordinary temperatures, were studied in absence of any catalyst.

Due to the possibility of formation of benzoquinones i.e. colored reaction products, the progress of reactions was followed by recording the absorbance of reaction mixture after different times at the λmax on Schimadzu double beam (UV-150-02) spectrophotometer. The reaction mixture of aniline, N-methylaniline (N-MA), N-ethylaniline (N-EA), N,N-dimethylaniline (DMA), 0-toluidine (0-TOL), m-toluidine (m-TOL), p-toluidine (p-TOL), N,N-dimethyl-p-toluidine (DMT), 0-ethylaniline (0-EA) p-ethylaniline (p-EA), 0-anisidine (0-ANIS), p-anisidine (p-ANIS), p-chloroaniline (p-CHL), p-bromoaniline (p-BRM) and p-phenatidine (p-PHEN) developed colors with λmax 355, 540, 480, 560, 525, 540, 470, 565, 490, 480, 500, 475, 530, 450, and 485 nm respectively.

Initial rates were evaluated in terms of change in absorbance with time i.e. $(dA/dt)_i$. The reactions were studied under pseudo first order conditions by taking one of the reactants in excess and the studies were kept confined to the period in which the λmax of reaction mixtures did not change.

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Guggenheim method was used for evaluating the first order rate constants and the second order rate constants were calculated by dividing the first order rate constant by the concentration of the reactant taken in excess. The second order rate constants were used for evaluation of kinetic parameters.

RESULTS AND DISCUSSION

Diffrent kinetic runs under pseudo first order conditions were carried out in the presence of different amounts of acetone ranging from 1.25% to 20% (v/v). The rate was found to decrease with decreasing dielectric constant (D) (Table- 1 to 10). A plot vetween $(dA/dt)_i$ and 1/D (Amis plot) [15] was found to be linear with negative slope suggesting thereby that the rate determining step is the reaction between a dipole and anion. Periodate ion exists as monoanion in the pH range 5-7[1-9]. So it is expected that aniline and periodate monoanion are the reacting species in the rate determining step.

The reactions were studied at four different temperatures ranging from 25°C to 45°C under pseudo first order conditions. Activation parameters were evaluated from the linear Arrhenius plot obtained by us. The mean values of energy of activation (Ea), frequency factor (A), enthalpy of activation ($\Delta H^{\#}$), entropy of activation ($\Delta S^{\#}$) and free energy of activation activation ($\Delta F^{\#}$), as evaluated from the values of second order rate constants, are given in the table-11.

A low value of activation energy is characteristic of bimolecular reactions in solution. A large negative value of entropy of activation observed by us is suggestive of the formation of a charged and rigid transition state which is expected to be strongly solvated in the polar solvent employed. This assumption is also sopported by the fact that the rate decereases with decreasing dieletric constant. Further, it definitely

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suggests the involvement of solvation effects in the mechanism of these reactions.

In addition to these studies, following observations were made by us:

(1) $[S]^{-1}$ vs $(\text{rate})^{-1}$ plot has been found to be a straight line passing through the origin suggesting thereby, that the intermediate formed in slow step of mechanism is unstable and undergoes the oxidation in a fast process [1]. Here S stands for aniline used.

(2) A plot of $\Delta S^{\#}$ and $\Delta H^{\#}$ for various substituted anilines was found to be linear giving a value of 273.27K for isokinetic temperature. It suggests the involvement of same interaction mechanism for the reaction series.

(3) A concave upward Hammett plot [16] indicates that electron donating substituents stabilize by resonance, a transition state having a high positive charge on the reaction centre which in this case obviously is the nitrogen of anilino group. This facilitates the bond breaking process in the transition state. On the other hand an electron withdrawing substituent increases the capacity of anilino nitrogen for a more negative charge in the transition state relative to the positive charge on the same nitrogen in ground state.

(4) A linear Brown and Okamoto plot [16] obtained by us reveals that the transition state should be with considerable positive charge.

(5) The main product of periodate oxidation of N-methylaniline was separated by us by allowing the reaction mixture standing overnight and extracting with petroleum ether and then separating a yellow compound on evaporation of ether. The m.p. of this compound was 115°C (lit. 115.7°C) [17]. It responded positively for quinone [18]. On the basis of m.p. and I.R. studies, it was characterised as p-benzoquinone.

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On the basis of kinetic studies discussed above and the product identified, the mechanism proposed for this class of reactions by taking N-methylaniline as a model is given in Chart-1.

In the proposed mechanism IO_4^- has been assumed as the oxidizing species in the light of the fact that the present studies were made by us at the pH 6.5 and IO_4^- is the reactive species in the pH range 5.0-7.0 [1-9].

The first step in the proposed mechanism is the bimolecular reaction between aniline and the periodate monoanion which is supported by the effect of solvent and high negative value of $\Delta S^{\#}$ observed by us. The formation of charged intermediate complex (I) by the attack of periodate ion on the nitrogen of anilino group and development of positive charge on nitrogen are supported by a non linear Hammett and a linear Brown and Okamoto plot. The high negative value of $\Delta S^{\#}$ and effect of dielectric constant support the involvement of solvation effects in these reactions. Intermediate (I) is unstable and undergoes the oxidation in a fast process- a fact supported by the observation that $[S]^{-1}$ vs $(rate)^{-1}$ plot is straight line passing through the origin. Intermediate (I) is likely to react with another molecule of periodate to form quinoneimines (II). The last step seems to be the fast hydrolysis of (II) to give benzoquinones (III).

On the basis of this mechanism and kinetic studies, the rate of reaction should be given by the relation

$$dx/dt = k_2 [S] [\text{periodate}]$$

This relation is in accordance with the second order behaviour and the stoichiometry observed for few reactions reported earlier [1-10].

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Table-1

[Aniline] = 5.0 x 10^{-3}M, [$NaIO_4$] = 5.0 x 10^{-4}M, Temp. = 40 ± 0.1°C
λmax = 355 nm, Medium = Methanol-water

Dielectric Constant (D)	69.60	68.90	64.13	54.82
$(dA/dt)_i$ x 10^3	3.6	3.0	1.5	0.4

Table-2

[N-MA] = 2.0 x 10^{-3} M, [$NaIO_4$] = 2.0 x 10^{-4} M, Temp. = 35 ± 0.1°C
λmax = 540nm, Medium = Acetone - water

D	73.9	72.4	70.0	66.8
$(dA/dt)_i$ x 10^3	5.2	4.0	3.0	2.5

Table-3

[N-EA] = 1.0 x 10^{-3} M, [$NaIO_4$] = 1.0 x 10^{-2} M, Temp. = 35 ± 0.1°C
λmax = 480nm, Medium = Acetone - water

D	73.9	72.4	70.0	66.8
$(dA/dt)^i$ x 10^3	3.2	2.75	2.2	1.5

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Table-4

[DMA] = 1.0 x 10^{-3} M, [$NaIO_4$] = 1.0 x 10^{-2} M, Temp. = 35 ± 0.1°C
λmax = 560nm, Medium = Acetone - water

D	74.6	72.4	70.0	66.8
$(dA/dt)^i$ x 10^3	3.1	2.6	2.1	1.3

Table-5

[0-TOL] = 1.0 x 10^{-2} M, [$NaIO_4$] = 1.0 x 10^{-3} M, Temp. = 25 ± 0.1°C
λmax = 525nm, Medium = Acetone - water

D	73.9	72.4	70.0	66.8
$(dA/dt)^i$ x 10^3	6.5	5.0	3.0	1.5

Table-6

[m-TOL] = 1.0 x 10^{-3} M, [$NaIO_4$] = 1.0 x 10^{-2} M, Temp. = 35 ± 0.1°C
λmax = 540nm, Medium = Acetone - water

D	73.9	72.4	70.0	66.8
$(dA/dt)^i$ x 10^3	8.25	7.0	3.5	2.0

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Table-7

[p-TOL] = 1.0 x 10^{-3} M, [$NaIO_4$] = 1.0 x 10^{-2} M, Temp. = 35 ± 0.1°C
λmax = 470nm, Medium = Acetone - water

D	73.9	72.4	70.0	66.8
$(dA/dt)_i$ x 10^3	12.0	8.0	5.0	4.0

Table-8

[DMT] = 1.0 x 10^{-3} M, [$NaIO_4$] = 1.0 x 10^{-2} M, Temp. = 35 ± 0.1°C
λmax = 565nm, Medium = Acetone - water

D	73.9	72.4	70.0	66.8
$(dA/dt)^i$ x 10^3.	16.1	12.0	7.0	3.9

Table-9

[o-EA] = 1.0 x 10^{-3} M, [$NaIO_4$] = 1.0 x 10^{-2} M, Temp. = 35 ± 0.1°C
λmax = 490nm, Medium = Acetone - water

D	73.9	72.4	70.0	60.8
$(dA/dt)_i$ x 10^3	5.5	4.25	3.5	2.5

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Table-10

[o-ANIS] = 1.0 x 10^{-3} M, [$NaIO_4$] = 1.0 x 10^{-2} M, Temp. = 35 ± 0.1°C
λmax = 500nm, Medium = Acetone - water

D	74.6	73.9	72.4	70.0	66.8
$(dA/dt)_i$ x 10^3	52.0	45.0	35.0	25.5	8.5

Table-11

Kinetic parameters for the periodate oxidation of different anilines in 10% (v/v) acetne-water medium

S.No.	Aniline	E_a (k cal mol^{-1})	$\Delta F^{\#}$ (k cal mol^{-1})	$\Delta S^{\#}$ (k cal mol^{-1})	$\Delta H^{\#}$ (k cal mol^{-1})
1.	Aniline	8.66	20.86	-41.34	8.04
2.	N-MA	10.11	19.66	-32.17	9.48
3.	N-EA	10.06	19.97	-33.91	9.44
4.	DMA*	9.4	19.49	-33.87	8.90
5.	o-Tol	5.49	20.20	-49.66	4.78
6.	m-Tol	8.46	20.45	-52.30	7.86
7.	p-Tol	8.64	19.58	-36.94	8.03
8.	DMT*	4.73	18.90	-47.61	4.11
9.	o-EA	3.47	20.54	-57.57	2.70
10.	p-EA	5.68	20.94	-51.18	5.06
11.	o-ANIS	3.39	19.46	-53.77	2.78
12.	p-ANIS	4.21	19.44	-50.19	3.60
13.	p-CHL	4.33	20.63	-54.53	3.71
14.	p-BRM	7.41	21.31	-46.79	6.78
15.	p-PHEN	7.42	19.19	-39.88	6.81

* Percentage of acetone (v/v) is 1.265%, 15% and 20% for DMA, DMT and O-ANIS respectively.

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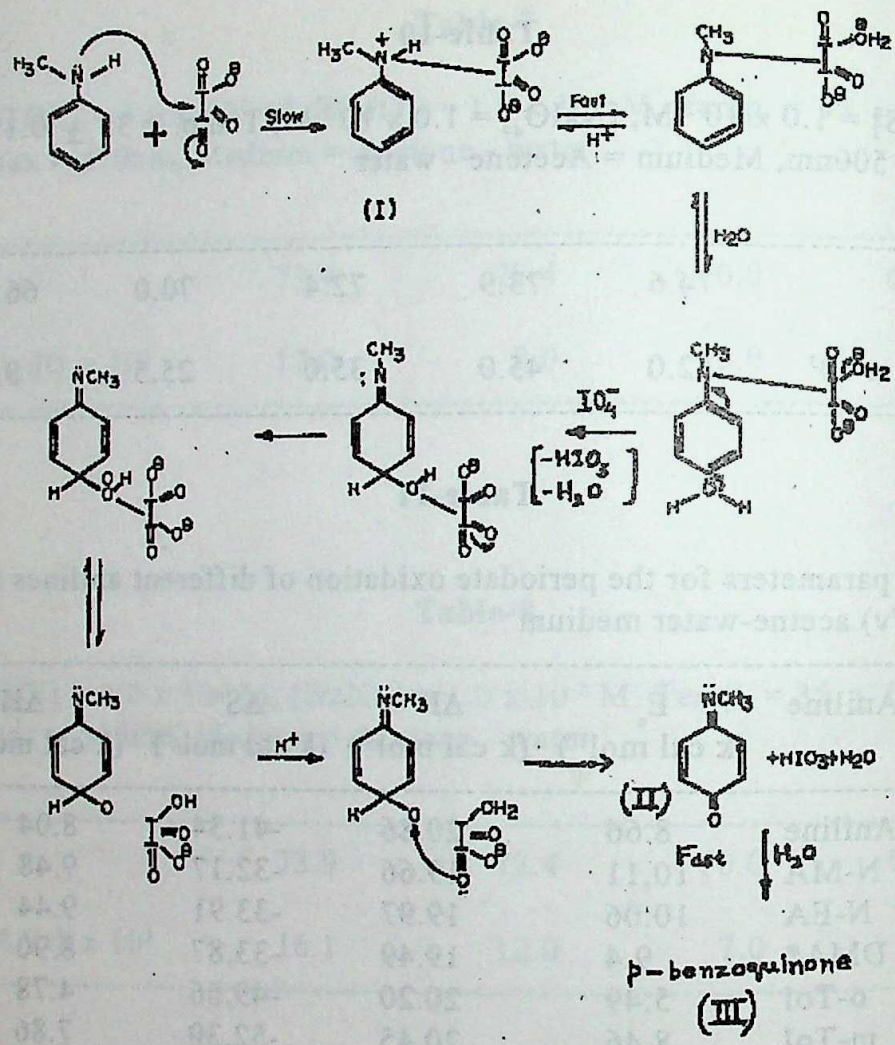

REFERENCES

1. G. Dryhurst: Periodate oxidation of diols and other functional groups, analytical structural applications, Pergamon Press, 1970.

2. V.K. Gupta, R.Bhushan, R.D.Kaushik, M.C.Jain and S.P.Srivastava: Kinetics and mechanism of the oxidation of aromatic amines by periodate ion-oxidation of aniline and N,N-dimethylaniline, Oxidation communications, 7(3-4) (1984), 409-423.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

3. S.P.Srivastava, M.C.Jain and R.D.Kaushik: Kinetics of periodate oxidation of aromatic amines, Nat. Acad. Sci. Letters, 2(2) (1970), 63-64.

4. M.P.Rao, B.Sethuram and T.N.Rao: Kinetics of oxidation of aniline and substituted anilines by periodate, Indian J.Chem., 17A (1979), 52-54.

5. S.P. Sivastava, G.Bhattacharjee and P.Malik: Periodate oxidation of 2,5-dimethylaniline in aqueous acetic acid- A kinetic and mechanistic study, Indian J.Chem., 25A (1986), 48-50.

6. R.D.Kaushik and Rajesh Joshi: Kinetics of Periodate oxidation of aromatic amines- A comparision of effect of pH on oxidation of some anilines, Asian J.Chem., 9(4) (1997), 742-745.

7. V.K.Pavolva, Ya.S.Sevchenko and K.B. Yatsmiriskii: Kinetics and mechanism of oxidation of diethylaniline by periodate, Zh.Fiz. Khim. 44(3) (1970), 658-63.

8. S.P.Srivastava, G.Bhattacharjee, V.K.Gupta and Satya pal: Kinetics and mechanism of oxidation of sulfanilic acid by periodate ion in aqueous medium, React. Kinet. Catal. Lett., 13(3) (1980), 231-237.

9. R.D.Kaushik and Rajesh Joshi: Kinetics and mechanism of periodate oxidation of m-toluidine in acetone-water medium, Asian J.Chem. 9(3) (1997), 527-532.

10. V.E.Kalinina: Kinetics and mechanism of periodate oxidation reactions catalysed by Ruthenium compounds, Kinet. Katal., 12(1) (1971), 100-106.

11. H.Tanabe: Periodic acid oxidation of N-glycosides, IX, periodic acid oxidation of aniline (2), Yokugaku Zasshi, 77(1957), 161-165.

12. H.Tanabe: Periodic acid oxidation of N-glycosides, XI, periodic acid oxidation of aniline (4), Yokugaku Zasshi, 78 (1957), 410-414.

13. H.Tanabe: Periodic acid oxidation of N-glycosides, XIII, periodic acid oxidation of aniline (6), Chem. Pharm. Bull., 7(1959), 177-83.

14. H.T.S.Britton: Hydrogen ions, D.Von Nostrand Co., (1956), 40.

15. E.S.Amis: Solvend effects on reaction rates and mechanism, Academic press (N.Y.), (1966), 42.

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16. R.D.Kaushik, Rajesh Joshi and Dheer Singh: Periodate oxidation of aromatic amines: Studies on the role of substituents and linear free energy relationahips, Asian J.Chem. 10(3) (1998), 567-572.

17. J.H. Dean: Lange's hand book of Chemistry, 13th ed., Mc Graw-Hill Co., (1985), 7.134.

18. A.I.Vogel: A text book of practical organic chemistry, Longman (London), (1957), 747.

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका, खण्ड 12 (1998) अंक 2, 47-70
Journal of Natural & Physical Sciences, Vol. 12 (1998) No.2, 47-70

REALIZATIONS OF INFINITESIMAL GENERATORS OF COMPLEX ANGULAR MOMENTUM OPERATORS

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(Received 28.12.98)

ABSTRACT

Realizations of complex momentum operators have been obtained for timelike, lightlike and spacelke cases and their explicit expressions have been derived under the Chakrabarti and Coester transformations. It has been shown that all the realizations for discrete-spin massless follow in the limiting case from the corresponding realizations for non-zero mass and imaginary mass cases. It has also been shown that the determination of any realization of the continuous-spin massless case automatically gives, as a special case, the corresponding realizations for the discrete-spin case. The specific discrete-spin and all continuous-spin realizations of massless case have been shown to be expressible in terms of Foldy-Shirokov forms while for spacelike case such connection has been shown only for the representations of little group SU (1,1) belonging to continuous series.

Kewwords : Complex angular momentum operator, generator, representation, transformation.

INTRODUCTION

In the history of diversified ways of the study of poincare group after the pioneer work of Wigner[1], one of the quite convcnient methods have been to put the emphasis on the homogeneous Lorentz group rather than on the translation group. It leads to the study of the Pioncare irreducible unitary representations in the Lorentz basis[2-6]. In last few years there

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has been considerable interest in the unitary irreducible representations of the homogeneous Lorentz group in various bases[7-15]. Harmonic analysis of a scaler function in terms of the four subgroup bases, i.e., 0(3), 0(2,1) E(2) and 0(1,1) (X) 0(2), has first been given by Vilenkin *et al*[7]. Most of the attention has been paid to the little group bases as these also play a role in the usual Pioncare invariant partial wave analysis[8,10,16,17,18] of scalar function and helicity amplitudes. Some subgroups of the homogeneous Lorentz group have been pointed out as subgroup type bases and non-subgroup type bases. The representation theory of the homogeneous Lorentz group or the covering group SL (2,C), is of great importance in physics and the knowledge of its Clebsch Gordan Coefficients is of fundamental importance in various areas of elementary particle physics because in addition to its extensive use in scattering theory and the theory of wave equations[11], it serves as the group of motions of space independent kinematical variable[19], as the invariant group of zero angle elastic scattering amplitude[9] and also as the group generating a complete set of basis functions[17].

Recent developments of the theory of particles moving faster than ligh[20] and the realizations in six-dimensional space-time can be described in most convenient way in terms of representations of this group. These representations are strictly connected to the represnetations of little group corresponding to a spacelike four-mementum. The representations for the timelike and lightlike cases have found an application in relativistic elementary particle theory, as these representations are assumed to describe the relativistic quantum-mechanical behaviour of stable and metastable particles with non-zero real mass and zero-mass quanta like photons and neutrinos. The representations corresponding to spacelike case have been making their appearance recently in several investigations, some of a physical and some of a mathematical nature. On the

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physical side, the proper physical interpretation of the propagation of spacelike particles (tachyons) if they exists, as well as some aspects of their interactions, have been discussed[21]. On the mathematical side, the spacelike representations have been encountered in several infinitic-component relativistic wave equations as well as in the work on the representations of current algebra in the infinite momentum limit.

The systematic treatment for the transformation matrix elements between different types of basis sets in SL (2, C) representations space and for the boost matrix elements have beengiven by Szego and Toth[22], Bargmann[23], Gelfand-Naimark[24], Toller[25], Nambu[26] and Bars-Gursey[27]. In our earlier papers[28,29,30], we have developed a compact operator formulation to reformulate the Gelfand-Naimark[24] theory of representations of SL (2, C) group by combining the generators of homogeneous Lorentz group. Diagonal elements of the operators thus obtained, have been interpreted as the components $\hat{Z}_1$, $\hat{Z}_2$ and $\hat{Z}_3$ of complex angular momentum operator $\hat{Z}$ in a complex space. Representations of complex angular momentum operator $\hat{Z}$ have been derieved in the basis of eigenvectors of third component of ordinary angular momentum operator and it has been shown that the space R in which the generators of homogeneous Lorentz group operate may be analyzed into a linear sum of invariant subspace R_1 in each of which an irreducible representation of weight '1' of the group of rotations is obtained. It has also been shown the eigenvectors of the generators of complex angular momentum operator form the complete set in the subspace in which only the weight '1' participates in the representations while in the subspaces in which weights '1-1' and '1+1' participates separately, they do not form the complete set[29]. The expression for matrix elements corresponding to finite complex rotations have been derived in terms of the complex Euler angles of the complex rotations. Using this expression

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the values of matrix elements for special cases of eigenvalues 1,2 and 3 of complex angular momentum operators have been computed in complex space corresponding to the values of the spin $\frac{1}{2}$, 1, $\frac{3}{2}$ in real space[30].

In the present paper the realizations of complex angular momentum operators have been derived for timelike. lightlike and spacelike cases corresponding to particles with real non-zero mass, zero mass and imaginary mass systems respectively and for all these cases their explict expressions have been derived under the transformations of Chakrabarti[31] and Coester[32]. It has been shown that the components of linear momentum in these expressions remain unchanged, while the components of spin matrices are modified. It has also been demonstrated that all the realizations for discrete-spin massless case follow from the corresponding realizations for non-zero mass case and imaginary mass case in the limit $m \to 0$. It has also been shown htat the determination of any realizations of the continuous-spin massless case automatically gives as a spicial case the corresponding realizations for the discrete-spin case. It has been clearly demonstrated that the study of zero-mass particles is directly connected with the non-zero mass case and their realizations in the specific discrete-spin and all continuous-spin forms may be considered in terms of Foldy-Shirokov standard forms. The equivalence of realizations of complex angular momentum operator for continuous-spin massless case and the Foldy-Shirokov forms has been emphasized.

It has also been shown that for the spin-zero particles, the realizations of complex angular momentum operators for timelike, spacelike and lightlike cases are all correspondingly identical. It is concluded that the space-like realizations of complex angular momentum operators for continuous-spin case are identical to Foldy-Shirokov form of non-zero real mass case if the representations of little group SU(1,1) belong to the continuous series.

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COMPLEX ANGULAR MOMENTUM OPERATORS IN TERMS OF MOMENTUM REPRESENTATION.

In our previous paper[28], we have combined the generators of homogeneous Lorentz group to define the components of complex angular momentum operator in the following from;

$$\hat{Z}_j = \frac{1}{2}(\hat{J}_j + i\hat{K}_j) \tag{1}$$

and

$$\hat{X}_j = \frac{1}{2}(\hat{J}_j - i\hat{K}_j) \tag{2}$$

where $\hat{J}_1, \hat{J}_2, \hat{J}_3$ are the generators corresponding to the ordinary angular momentum operators associated with rotations and $\hat{K}_1, \hat{K}_2, \hat{K}_3$ are the generators for pure Lorentz transformations. These components of complex angular momentum operator have been shown to satisfy following communication rules :

$$\left[\hat{Z}_j, \hat{Z}_k\right] = i \in_{jkn} \hat{Z}_n, \tag{3}$$

$$\left[\hat{X}_j, \hat{X}_k\right] = i \in_{jkn} \hat{X}_n, \tag{4}$$

$$\left[\hat{Z}_j, \hat{X}_k\right] = 0, (j,k,n = 1,2,3) \tag{5}$$

where ε_{jkn} is the usual Levi-Civita three index symbol and the summation convention is employed for repeated indices. Let us now consider separately the realizations of these operators for timelike, lightlike and spacelike cases[1] of irreducible unitary representations of Poincare group corresponding to particles with non-zero real mass, zero-mass and imaginary mass respectively, for which the Casimir operators corresponding to four-momenta operator and the operator associated with the

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Pauli-Lubanski four-vector generate the Wigner's little groups SO(3), E(2) and SO(2,1) respectively[33].

Non-Zero Real Mass Case

For such cases the little group generators are three spin operators generating a SO (3) structure homomorphic to SU (2), and the famous Foldy[34] - Shirokov[35] realization [the F-S forms] of the generators of homogeneous Lorentz group $\hat{J}$ and $\hat{K}$ is very well known[36-38]. In the momentum representation space this F-S form is given by

$$\hat{J} = -i\,\vec{p} \times \frac{\partial}{\partial \vec{p}} + \vec{S} \tag{6a}$$

$$\hat{K} = -i\,w \times \frac{\partial}{\partial \vec{p}} - \frac{\vec{S} \times \vec{p}}{w+m} \tag{6b}$$

where $w=(m^2 + p^2)^{1/2}$ and S1, S2, S3 represent three irreducible (2s+1) dimensional matrices of arbitrary spin s such that

$$\left[S_j, S_k\right] = i \in_{jkn} S_n \tag{7}$$

In the representation given by Eq. (6) there is no mixing of spin and momentum under a rotation in space and the angular momentum operators are the sums of two kinematically independent parts, the orbital and spin angular momenta. For timelike case this F-S form always exists whatever be the representation of the little group SU(2) that is used.

Using Eqs. (6) we may have the following F-S form for the complex angular momentum operator,

$$\hat{Z}_T = -i\,\vec{p} \times \tilde{\nabla} + \tilde{S}. \tag{8}$$

(T denotes timelike case)

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Which is the similar form as that given by Eqs. (6) for ordinary angular momentum operator except that operators $\vec{\nabla}$ and $\vec{S}$ are replaced by operators $\tilde{\nabla}$ and $\tilde{S}$ given by

$$\tilde{\nabla}=\frac{1}{2}\left(\vec{\nabla}-\frac{\vec{S}}{w+m}\right) \tag{9a}$$

and

$$\tilde{S}=\frac{1}{2}\left(\vec{S}-w\vec{\nabla}\right), \tag{9b}$$

where $\vec{\nabla}=\dfrac{\partial}{\partial\vec{p}}$.

Eqs. (9) represents a kinematical coupling between orbital motion and internal rotation of a relativistic rotator. If the tetrapod decribing kinematically a relativistic rotator is operated upon by the operator $\hat{Z}_T$ given by (8), it may be demonstrated that the rotator will have an accelerated orbital motion even for a free system.

Let us now apply the following unitary transformation of Chakrabarti[31] to the generators given by Eq. (8);

$$U_1=\exp\left[i\theta\,\frac{p_1S_2-p_2S_1}{\left(p_1^2+p_2^2\right)^{1/2}}\right], \tag{10}$$

where $\theta=\text{arc}\,tg\dfrac{\left(p_1^2+p_2^2\right)^{1/2}}{p_3}$ $i.e.,\ \theta=\cos^{-1}\dfrac{p_3}{p}$ (10a)

This transformation diagomalizes the helicity operator

$$U_1\frac{\vec{S}.\vec{p}}{|\vec{p}|}U_1^{+}=S_3 \tag{11}$$

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and the transformed generators are

$$\hat{Z}_{Tj}^{(1)} = -\frac{1}{2}\epsilon_{jnk}\, p_n \left[\frac{\partial}{\partial p_k} - \frac{w}{p^2} S_3 X_k\right] - \frac{p}{2}\left[\frac{w}{p}\frac{\partial}{\partial p_j} - \frac{S_3}{p} X_j\right]$$

$$-\frac{im}{2p^2}(\vec{p}\times\vec{S})_3 X_j + \frac{im}{2p} e_{3jn} S_n, \tag{12}$$

where $\vec{X} = \begin{pmatrix} \dfrac{p_1}{p+p_3} \\ \dfrac{p_2}{p+p_3} \\ 1 \end{pmatrix}$ and $\vec{e}_3 = (0,0,1)$ (13)

Eq. (12) shows that under the transformation with unitary matrix U_1 given by Eq. (10) the components of linear momentum remain unchanged in the expression of the generator $\hat{Z}_j$ $(j = 1,2,3)$ while the matrices S_j are modified by the components of matrix $\vec{X}$. Similarly, under Coester transformation[32] with unitary matrix

$$U_2 = \exp(i\theta S_2)\exp(i\phi S_3), \tag{14}$$

where polar angle θ has beendefinied by Eq. (10a) and ϕ is given by

$$\phi = \tan^{-1}\frac{p_2}{p_1}. \tag{14a}$$

This transformation diagonalizes the helicity operator

$$U_2 \frac{\vec{S}.\vec{p}}{|\vec{p}|} U_2^{+} = S_3 \tag{15}$$

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and gives the following transformation of the generator $\hat{Z}_T$,

$$\hat{Z}_{Tj}^{(2)} = -\frac{i}{2}\in_{jnk} p_n\left[\frac{\partial}{\partial p_k}-\frac{w}{p}S_3 Y_k\right]-\frac{p}{2}\left[\frac{w}{p}\frac{\partial}{\partial p_j}-S_3 Y_j\right]$$

$$+iS_1 m\frac{(p_1^2+p_2^2)^{1/2}}{2pp_3}\in_{jnk} p_n Y_k+\frac{i S_2 m p_3}{2p^2}(p_1^2+p_2^2)^{1/2}\left(Y_j-\frac{S_{3j}}{p_3}\right) \quad (16)$$

where

$$\vec{Y}=\frac{1}{p_1^2+p_2^2}\begin{pmatrix} p_1 \\ p_2 \\ 0 \end{pmatrix} \quad (17)$$

Zero-mass Case

For zero-mass case (lightlike case), we may more conveniently make the more natural choice of the following operators as the basis of Lie-algebra of the homogeneous Lorentz group instead of using $\vec{J}_k$ and $\vec{K}_k$,

$$\hat{J}_3;\hat{K}_3;\hat{T}_1 = \hat{K}_1-\hat{J}_2;\hat{T}_2 = \hat{K}_2+\hat{J}_1;$$

$$\hat{\eta}_1 = \hat{K}_1+\hat{J}_2;\hat{\eta}_2 = \hat{K}_2-\hat{J}_1 \quad (18)$$

where

$$\left[\hat{J}_3,\hat{T}_r\right]=i\varepsilon_{rs}\hat{T}_s;\left[\hat{J}_3,\hat{\eta}_r\right]=i\varepsilon_{rs}\hat{\eta}_s;$$

$$\left[\hat{K}_3,\hat{T}_r\right]=i\hat{T}_r;\left[\hat{K}_3,\eta_r\right]=i\hat{\eta}_r;$$

$$\left[\hat{J}_3,\hat{K}_3\right]=\left[\hat{T}_r,\hat{T}_s\right]=\left[\hat{\eta}_r,\hat{\eta}_s\right]=0; \quad (19)$$

$r,s=1,2$ and ε_{rs} is two-dimensional antisymmetric matrix with $\varepsilon_{12}=-\varepsilon_{21}=1$. The little group is generated by $\hat{J}_3,\hat{T}_1$ and $\hat{T}_2$ and has the structure of Euclidean group E (2). For this case the Casimir operator corresponding to four-momenta operator vanishes while the Casimir operator corresponding to Pauli-Lubanski four-vector is zero for discrete-spin representation and non-zero

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$\left(T_1^2 + T_2^2 \neq 0\right)$ for continuous-spin representation. The former one (i.e. discrete-spin representation) is more familiar in its use to describe particles like neutrinos and photons while the second one (i.e. continuous-spin representation) can be expressed in the famous S-F form[36]. Let us consider in the following the representations of complex angular momentum operators of lightlike case in these two subclasses separately.

Discrete-spin Case $\hat{T}_1 = \hat{T}_2 = 0$

For descrete-spin representation of lightlike case the complex angular momentum operator may be written in straight forward manner from Eq. (8) in the limit m→0. As such, we get

$$\hat{Z}_{LD} = -\frac{i}{2}\vec{p}\left(\vec{\nabla} - \frac{\vec{S}}{p}\right) - \frac{p}{2}\left(\vec{\nabla} - \frac{\vec{S}}{p}\right). \tag{20}$$

[LD denotes lightlike Discrete-spin case]

Under the Chakrabarti transformation carried by the unitary operator U_1 given by Eq. (10), the components of this complex angular momentum operator transformation in the following manner;

$$\hat{Z}^1_{LDj} = -\frac{i}{2}\in_{jnk} p_n\left(\frac{\partial}{\partial p_k} - \frac{S_3}{p}X_k\right) - \frac{p}{2}\left(\frac{\partial}{\partial p_j} - \frac{S_3}{p}X_j\right) \tag{21}$$

where vector $\vec{X}$ is given by Eq. (13). Similarly, under the Coester transformation given by unitary matrix U_2 given by Eq. (14) we get, the following transformations of the components of complex angular momentum operators;

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$$\hat{Z}^2_{LDj} = -\frac{i}{2} \in_{jnk} p_n \left(\frac{\partial}{\partial p_k} - S_3 Y_k \right) - \frac{p}{2} \left(\frac{\partial}{\partial p_j} - S_3 Y_j \right), \tag{22}$$

where vector $\vec{Y}$ is given by Eq. (17). Equations (21) and 922) show that under the transformations with unitary matrices U_1 and U_2 the components of linear momentum remain unchanged in the expression of the generator $\hat{Z}_j$ for discrete-spin lightlike case, while the matrices S_j are modified by the components of vectors $\vec{X}$ and $\vec{Y}$ respectively. The form of the generators given by equation (21) is similar to that proposed by Lomont and Moses[39] and the equation (22) corresponds to the generators proposed by Shirokov for (o.s) irreducible unitary representations[35] when the helicity is diagonalized. Equations (21) and (22) actually give the generators of complex angular momentum operators in the equivalent forms. This equivalence may be demonstrated by introducing an unitary transformation in terms of the unitary operator U_3 given by[40].

$$U_3 = \exp(-i\phi\ S_3) \tag{23}$$

where ϕ is given by equation (14a). Then we may get

$$U_1 = U_3 U_2 \tag{24}$$

which has a physical meaning that the transformation U_1 may be expressed as a real rotation[41] in spin space with Euler angles $(\phi, -\theta, -\phi)$ i.e.

$$U_1 = \exp(-i\phi S_3)\exp(i\theta S_2)\exp(i\phi S_3). \tag{25}$$

Continuous-Spin Case $\hat{T}_1^2 + \hat{T}_2^2 \neq 0$

For the continuous-spin case, there are one-valued and two-valued representations of the Pioncare group according to integer and the half

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integer eigen values of the helicity operator respectively. All these representations are infinite dimensional and for each one of them the F-S form exists. In some studies[42,43] these continuous-spin lightlike representations have been associated with continuous-spin particles and an extended V-A theory of weak interactions could be presented and related to the conventional theory. Corresponding to two solutions of Chakrabarti's construction[44] of representations in continuous-spin lightlike case, we get the following two forms of the complex angular momentum operator associated with continuous-spin particles of zero-mass;

$$\hat{Z}_{LC}^{(a)} = -i\,\vec{p}\times\tilde{\nabla}_a + \tilde{S} \tag{26}$$

and

$$\hat{Z}_{LC}^{(b)} = -i\,\vec{p}\times\tilde{\nabla}_b + \tilde{S} \tag{27}$$

where

$$\tilde{\nabla}a = \frac{1}{2}\left(\frac{\partial}{\partial\vec{p}} - \frac{\vec{S}}{p} - \frac{\vec{T}}{p^2}\right); \tag{28}$$

$$\tilde{\nabla}_b = \frac{1}{2}\left(\frac{\partial}{\partial\vec{p}} - \frac{\vec{S}}{p} - \frac{\vec{T}\times\vec{p}}{p^3}\right); \tag{29}$$

and

$$\tilde{S} = \frac{1}{2}\left(\vec{S} - p\frac{\partial}{\partial\vec{p}}\right). \tag{30}$$

(*Lc* in Eqs. (26) and (27) denotes lightlike continuous-spin). The symmetrical appearance of $\vec{S}$ in Eqs. (26) and (27) suggests the possibility of rotational symmetric appearance of the generators $\left(\vec{S},\vec{T}\right)$ of Euclidean group E(3). These

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Hermitian generators commuting with the orbital parts are characterised by the commutation relations;

$$\begin{aligned}
&\left[S_j, S_k\right] = i \in_{jkn} S_n; \\
&\left[S_j, S_k\right] = i \in_{jkn} T_n; \\
&\left[T_j, T_k\right] = 0,
\end{aligned} \tag{31}$$

including those of E(2) given by first and last constiuents of Eqs. (19). The two forms of complex angular momentum operator, given by Eqs. (26) and (27) are physically equivalent[45] constructions. Equivalence and realizations of different forms of these generators under different unitary transformation have been given by Beckers and Jaspers[46].

The transformation of the components of these operators under the Chakrabarti's operator U_1 given by (10) may be written in the following form;

$$\begin{aligned}
\hat{Z}^{(a)^1}_{LCj} &= -\frac{i}{2} \in_{jnk} p_n \left[\frac{\partial}{\partial p_k} - \frac{S_3}{p} X_k\right] - \frac{p}{2}\left[\frac{\partial}{\partial p_j} - \frac{S_3}{p} X_j\right] \\
&- \frac{i}{2p^2}\left(\vec{T} \times \vec{p}\right)_3 X_j - \frac{i}{2p} e_{3jn} T_n,
\end{aligned} \tag{32}$$

and

$$\begin{aligned}
\hat{Z}^{(b)^1}_{LCj} &= -\frac{i}{2} \in_{jnk} p_n \left[\frac{\partial}{\partial p_k} - \frac{S_3}{p} X_k\right] - \frac{p}{2}\left[\frac{\partial}{\partial p_j} - \frac{S_3}{p} X_j\right] \\
&- \frac{i}{2p^2}\left(\vec{p}.\vec{T}\right) X_j + \frac{i}{2p} T_j;
\end{aligned} \tag{33}$$

where $\vec{X}$ is givenby Eq. (13); $\vec{p}.\vec{T} = p_1 T_1 + p_2 T_2$ and $\vec{T} = \left(T_1, T_2, 0\right)$. (34)

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Similarly, the transformations of these operators under Coester's operator U_2 given by (14) may be derived in the following form;

$$\hat{Z}_{LCj}^{(a)(2)} = -\frac{i}{2}\in_{jnk} p_n\left[\frac{\partial}{\partial p_k} - S_3 Y_k\right] - \frac{p}{2}\left[\frac{\partial}{\partial p_j} - S_3 Y_j\right]$$

$$+\frac{iT_1}{2pp_3}\left(p_1^2+p_2^2\right)^{1/2}\in_{jnk} p_n Y_k$$

$$-\frac{iT_2\, p_3}{2p^2}\left(p_1^2+p_2^2\right)^{1/2}\left[Y_j - \frac{S_{3j}}{p_3}\right] \tag{35}$$

and

$$\hat{Z}_{LCj}^{(b)(2)} = -\frac{i}{2}\in_{jnk} p_n\left[\frac{\partial}{\partial p_k} - S_3 Y_k\right] - \frac{p}{2}\left[\frac{\partial}{\partial p_j} - S_3 Y_j\right]$$

$$+\frac{iT_2}{2pp_3}\left(p_1^2+p_2^2\right)^{1/2}\in_{jnk} p_n Y_k$$

$$+\frac{iT_1\, p_3}{2p^2}\left(p_1^2+p_2^2\right)^{1/2}\left[Y_j - \frac{S_{3j}}{p_3}\right] \tag{36}$$

where $\vec{Y}$ is given by (17).

Imaginary-Mass Case

A connection between complex angular momentum operator and the irreducible representation of the Poincare group corresponding to a spacelike four momentum has been well investigated[47,48,49]. These representations are connected to the representations of Wigner little group corresponding to a spacelike four momentum. In this case the little group

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is the three-dimensional homogeneous Lorentz group SO (2,1) which is homomorphic to the spinor group SU (1,1). Its generators $\hat{J}_3, \hat{R}_1 = -\hat{K}_2, \hat{R}_2 = \hat{K}_1$, and satisfy the commutation relations

$$\left[\hat{J}_3, \hat{R}_r\right] = i\,\varepsilon_{rs}\,\hat{R}_s.$$

$$\left[\hat{R}_r, \hat{R}_s\right] = -i\,\varepsilon_{rs}\,\hat{R}_3. \tag{37}$$

Representations of SO (2, 1) (and hence the spacelike ones of Poincasre group) are well known[23] and have been used on reduction and second quantization of fields associated with tachyons (spacelike particles or the particles with imaginary mass) of different spins[50, 51].

Using two solutions of Chakrabarti's[44] construction of representations for continuous-spin spacelike case we get the following two forms of the complex angular momentum operators associated with continuous-spin particles of imaginary mass system $(m = i\mu)$;

$$\hat{Z}_s^{(a)} = -i\,\vec{p} \times \tilde{\nabla}_a + \tilde{S}, \tag{38}$$

$$\hat{Z}_s^{(b)} = -i\,\vec{p} \times \tilde{\nabla}_b + \tilde{S}, \tag{39}$$

where

$$\tilde{\nabla}_a = \frac{1}{2}\left[\frac{\partial}{\partial \vec{p}} - \frac{w\vec{S}}{|\vec{p}|^2} + \frac{i\mu\vec{R}}{|\vec{p}|^2}\right]; \tag{40}$$

$$\tilde{\nabla}_b = \frac{1}{2}\left[\frac{\partial}{\partial \vec{p}} - \frac{w\vec{S}}{|\vec{p}|^2} + \frac{i\mu\left(\vec{R} \times \vec{p}\right)}{|\vec{p}|^3}\right]; \tag{41}$$

$\tilde{S}$ is defined by equation (9b) and the subscript S denoting the spacelike case with the energy cigen-values w given by

$$w = \left(|\vec{p}|^2 - \mu^2\right)^{1/2} \tag{42}$$

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reality of which requires

$$|\vec{p}| \geq \mu \tag{43}$$

In Eqs. (38-42) the generators $\left(\hat{S}, \hat{R}\right)$ of SO (3, 1) appear in a rotationally symmetric manner. These generators commute with the orbital parts and satisfy the commutation relations;

$$\begin{aligned}
\left[S_j, S_k\right] &= i \in_{jkn} S_n; \\
\left[S_j, R_k\right] &= i \in_{jkn} R_n, \\
\left[R_j, R_k\right] &= -i \in_{jkn} S_n,
\end{aligned} \tag{44}$$

including those of SO (2,1) given by (37).

Transformation of the components of complex angular momentum operator given by (38) and (39) for two different solutions of Chakrabarti, under the operator U_1 may be written in following forms respectively;

$$\begin{aligned}
\hat{Z}_{Sj}^{(a)(1)} &= -\frac{i}{2} \in_{jnk} p_n \left[\frac{\partial}{\partial p_k} - \frac{w S_3}{p^2} X_k\right] \\
&- \frac{p}{2}\left[\frac{w}{p}\frac{\partial}{\partial pj} - \frac{S_3}{p} X_j\right] - \frac{i\mu}{2p^2}\left(\vec{R} \times \vec{p}\right)_3 X_j \\
&- \frac{i\mu}{2p} e_{3jn} R_n
\end{aligned} \tag{45}$$

and

$$\begin{aligned}
\hat{Z}_{Sj}^{(b)(1)} &= -\frac{i}{2} \in_{jnk} p_n \left[\frac{\partial}{\partial p_k} - \frac{w S_3}{p^2} X_k\right] \\
&- \frac{p}{2}\left[\frac{w}{p}\frac{\partial}{\partial pj} - \frac{S_3}{p} X_j\right] - \frac{i\mu}{2p^2}\left(\vec{p} \cdot \vec{R}\right) X_j + \frac{i\mu}{2p}\vec{R},
\end{aligned} \tag{46}$$

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where

$$\vec{p}.\vec{R} = p_1 R_1 + p_2 R_2 \tag{47}$$

$$\vec{R} = (R_1, R_2, O) \tag{48}$$

and $\vec{X}$ is given by (13). Similarly, the transformations of the components of complex angular momentum operators under the operator U_2 may be derived in the following form;

$$\hat{Z}_{Sj}^{(a)(2)} = -\frac{i}{2} \in_{jnk} p_n \left[\frac{\partial}{\partial p_k} - \frac{w}{p} S_3 Y_k \right] - \frac{p}{2} \left[\frac{w}{p} \frac{\partial}{\partial p_j} - S_3 Y_j \right]$$

$$+ \frac{i \mu R_1}{2 p p_3} \left(p_1^2 + p_2^2 \right)^{1/2} \in_{jnk} p_n Y_k$$

$$- \frac{i \mu R_2 p_3}{2 p^2} \left(p_1^2 + p_2^2 \right) \left[Y_j - \frac{S_{3j}}{p_3} \right] \tag{49}$$

and

$$\hat{Z}_{Sj}^{(b)(2)} = -\frac{i}{2} \in_{jnk} p_n \left[\frac{\partial}{\partial p_k} - \frac{w}{p} S_3 Y_k \right] - \frac{p}{2} \left[\frac{w}{p} \frac{\partial}{\partial p_j} - S_3 Y_j \right]$$

$$+ \frac{i \mu R_2}{2 p^2 p_3} \left(p_1^2 + p_2^2 \right)^{1/2} \in_{jnk} p_n Y_k$$

$$+ \frac{i \mu p_3 R_1}{2 p^2} \left(p_1^2 + p_2^2 \right)^{1/2} \left[Y_j - \frac{S_{3j}}{p_3} \right] \tag{50}$$

where $\vec{Y}$ is given by (17). It is obvious from (45), (46), (49) and (50) that the above two sets of spacelike realizations of complex angular momentum operators are not mutually independent, rather a sort of extraconnections hold between them.

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DISCUSSION

The realizations of complex angular momentum operator for timelike, lightlike and spacelike cases have been obtained in equations [18, 26, 27, 36 and 37] respectively, by combining the generators of proper, orthochronous, homogeneous Lorentz group without including the space and time reflections. These equations represent the kinematical coupling between orbital motion and internal rotation of a relativistic rotator and obviously lead to the conclusion that if the tetrapode describing kinematically a relativistic rotator is operated upon by the generators of complex angular momentum operator then the rotator will have an accelerated orbital motion even for a free system. Equation (8) gives the realization of complex angular momentum operator for timelike case (non-zero real mass system) in the Foldy-Shirokov form which is similar to that given by equation (6a) for ordinary angular momentum operator except that the operator $\vec{\nabla}$ has been replaced by $\tilde{\nabla}$ given by equation (9a) and $\vec{S}$ has been replaced by $\tilde{S}$ given by equation (9b). As such, the kinematical independence between the two parts corresponding to orbital and spin angular momenta in equation (8) is not that strict. The symmetrical appearance of $\vec{S}$ and $\vec{T}$ in equations (26) and (27) of generators of complex angular momentum operators for the two solutions for continuous-spin lightlike case, suggests the possibility of rotational symmetric appearance of the generators of Euclidean group E (2). Similarly, the generators $\vec{S}$ and $\vec{R}$ of the group SO (2,1) appear in rotationally symmetric manner in equations (36) and (37) which give the realizations of complex angular momentum operator for the two solutions in continuous-spin spacelike case. Equations (12) and (16) give the realizations of complex angular momentum operator for non-zero real mass system under the Chakrabarti's[31] and Coester's[32] unitary transformations respectively. These equations show that under these unitary transformations carried by unitary matrices U_1 and U_2 given by

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equations (10) and (14) respectively, the components of linear momentum remain unchanged in the expressions of the generators of complex angular momentum operator while the matrices S_j are modified by the components of matrices $\vec{X}$ and $\vec{Y}$ given by equations (13) and (17) respectively. Transformation equations (21) and (22) have been derived by applying the unitary transformations given by equations (10) and (14) upon the operators $\hat{Z}_{LD}$ given by equation (20) for the complex angular momentum in discrete-spin lightlike case. These equations also follow from equations (12) and (16) respectively, for timelike case in the limit $m \to 0$. Such a connection between descrete-spin representations (m,s) and (o,s) for timelike and lightlike cases respectively through the limiting process $m \to 0$ is an interesting one and has already applied to the discussions on vector meson dominance in photo production and the corresponding T-matrix elements. Though the realizations (21) and (22) of complex angular momentum operator under Chakrabarti's and Coester's transformation here are different since Chakrabarti's realizations give the Lomont-Moses[39] result while those of Coester give Shirokov[35] realizations, they are physically equivalent.

The two sets $\hat{Z}^{(a)}$ and $\hat{Z}^{(b)}$ of complex angular momentum operators, for continuous-spin representations given by the sets of equations (26, 32, 35) and (28) also (27, 33, 36) and (29) respectively, are not independent from each other. Rather they are related to each other by unitary transformations. A comparison of realizations given by equations (12) and (16) for non-zero real mass case with the corresponding realizations of $\hat{Z}^{(a)}$ given by (32) and (35) for continuous-spin massless case shows an excellent similarity leading to the conclusion that $\hat{Z}^{(a)}$ realizations of complex angular momentum operator is identical to Foldy-Shirokov form for non-zero mass case while such similarity does not exist for $\hat{Z}^{(b)}$. But due to the existence of equivalence

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between $\hat{Z}^{(a)}$ and $\hat{Z}^{(b)}$ realizations, it may be possible to cast later ones also into a Foldy-Shirokov form. These conclusions give a modification, without contradicting, of the results of Mukunda[36]. $\hat{Z}^{(a)}$ and $\hat{Z}^{(b)}$ realizations for spacelike case given by equations (38), (45), (49) and (39), (46), (50) are similar in nature to those of continuous spin lightlike case given by equations (26), (32), (35) and (27), (33), (36) except that in place of operators $\hat{T}_1$ and $\hat{T}_2$ in later case we have $\hat{R}_1$ and $\hat{R}_2$ in the former case in addition to the occurence of imaginary mass.

A comparison of realizations given by set of equations (8), (12), and (16) for real non-zero mass case with the corresponding realizations $\hat{Z}^{(a)}$ given by set of equations (38), (39), (45), (46), (49) and (50) for continuous-spin spacelike case (i.e. imaginary mass system) shows an excellent similarity. They are rather identical except that S_1 and S_2 in the former set are replaced by $(-\hat{R}_1)$ and $(-\hat{R}_2)$ in the later one and that the every eigenvalues $w = (m^2 + p^2)^{1/2}$ of the former case are replaced by those given by equation (42) in the later case. This similarity, leads to the conclusion that $\hat{Z}^{(a)}$ spacelike realizations of complex angular momentum operator for continuous-spin case is identical to Foldy-Shirokov form of non-zero mass case if the representation of little group SU (1, 1) belong to continuous series.

In order to get the corresponding realization for discrete-spin case we shall undertake the standard helicity representation for these operators in the second part.

REFERENCES

1. E.P. Wigner : On Unitary representations of the inhomogeneous Lorentz group, Ann. Math. 40(1939), 149.

2. I.S. Shapiro : Expansion of a wave function in irreducible representations of the Lorentz group, Dokl. Adad. Nauk, 106(1956), 647.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

3. G. Feldman and P.T. Mathews : Poincare irreducible unitary representations in a Lorentz basis, Ann. Phys. 55(1969), 506.

4. A Chakrabarti. M. Levy-Nahas and R. Seneer. Lorentz basis of the poincare group, J. Math. Phys. 9(1968), 1274.

5. A Chakrabarti : Partial wave expansions of scattering amplitudes of irreducible unitary representations in the Lorentz basis, J. Math. Phys. 12(1971), 1822.

6. B. Radhakrishnan and N. Mukunda : The study of Poincare irreducible unitary representations in a Lorentz basis, J. Math. Phys. 15 (1972), 477

7. N. Y. Vilenkin and Y.A. Smorodinski : Invariance expansions of relativistic amplitudes, Soviet Physics JETP, 19 (1964), 1209.

8. P. Wintermitz, Y.A. Smorodinski and M.B. Sheptel : Poincare and Lorentz-Invariance expansions of relativistic amplitudes.

9. A Sciarrino and M. Toller : Decomposition of the Unitary irreducible representations of the group SL(2,C) restricted to the subgroup SU(1,1), J. Math. Phys. 8(1967), 1252.

10. R. Delbourgo, A Salam and J. Strathedee : Harmonic group, Analysis in terms of the homogeneous Lorentz group, J. Phys. Lett. 25B (1967), 230.

11. N. Mukunda : Unitary representations of the Lorentz groups, Reduction of the supplementary series under a noncompact subgroup, J. Math. Phys. 9(1968), 417.

12. N.W. Macfadyen : The reduction $0(3,1) \supset 0(2,1) \supset 0(1,1)$ J. Math. Phys. 12(1971), 492.

13. I. Lukac, Y.A. smorodinski and P. Winternitz : Quantum numbers in the little groups of the poincare group, Sov. J. Nucl. Phys. 7(1968), 139.

14. S. Chang and L.O'Raifeartaigh : Unitary representations of SL (2,C) in an E(2) basis, J. Math. Phys. 10 (1969), 21.

15. Y.A. Smorodinski and M. Huszar : Representations of Lorentz Group, Theor. Math. Phys. 4 (1970), 867.

16. H. Joss : Zur Darstellungs Theorie der inhomogenen Lorentz gruppe als grundlage quantenniechanischer Kinematic, Fortschr. d. Phys. 10 (1962), 65.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

17. J. F. Boyce, R. Delbourgo. A. Salarm and J. Strathdee : The Poincare group generating a complete set of basis functions, J. Ic/67/69, Trieste (1967).

18. E.G. Kalnins, J. Patera, R.T. Shart and P. Winternitz : Group Theory and It's Applications (E.M. Loebl. Ed.), vol. III, Academic Press, New York (1975), 369.

19. C. Fronsdal : Jomogeneous Lorentz group of motions of space independent kinematical variable, Phys. Rev. 156(1967), 1653.

20. B.S. Rajput and K.D. Purohit : Study of Tachyonic Lorentz transformations. Ind. J. Phys. 54A(1980), 377.

21. E.C.G. Sudarshan : The theory of particles travelling faster than light-I, Syracuse University preprint NYO-3399-186, Dec. 1968.

22. K. Szego and K. Toth : SL (2,C) representations in explicitly energy dependent basis, J. Math. Phys. 12(1971), 846.

23. V. Bargann : Irreducible Unitary representations of the Lorentz group, Ann. of Math., 48 (1947), 568.

24. I.M. Gel'ford and M.A. Naimark : Unitary representations of the Lorentz group. Izv. Akad. Nauk. SSSR Set. Mat., 11(1947), 411.

25. M. Toller : An expansion of the soattering amplitude at vanishing four-momentum transfer using the representations of the Lorentz group, Nuovo Cimento, 53(1968), 671.

26. Y. Nambu : SL(2,C) representations of Lorentz group, Phys. Rev. 160(1967), 1171.

27. I. Bars and F. Guosey : Representations of SL(2, C) group, J. Math. Phys. 13(1972), 131.

28. K.N. Joshi and B.S. Rajput : Complex angular momentum operators in terms of generators of homogeneous Lorentz group. Proceedings : Einstein Centenary Symposium (1980), 431.

29. K.N. Joshi and B.S. Rajput : Addition of complex angular momentum operators, J. Math. Phys. 21(1981), 1579.

30. K.N. Joshi and B.S. Rajput : Irreducible representations of the complex rotation group in terms of the axix and angle of rotations, J. Math. Phys. 22(1981), 2486.

31. A Chakrabarti : United Unitary representation of the Poincare group for particle of zero and positive rest mass. J. Math. Phys. 7(1966), 949.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

32. F. Coester : Particles with zero mass and particles with "small" mass, Phys. Rev; 129 (1963), 2816.

33. J. Strathdee, J.F. Boyce, R. Delbourgo and A. Salam : Partial Wave analysis, Part I, IAEA/67/69, Trieste, 1967.

34. L.L. Foldy : Synthesis of covariant particle equations, Phys. Rev. 102(1956), 568.

35. I.M. Shirokov : A group theoretical consideration of the basis of relativistic quantu machanics I, II, III. The general properties of the inhomogeneous Lorentz group. Sov. Phys, JETP 6(1958), 664, 919, 929.

36. N. Mukunda : Photons and Tachyons with continuous spin, Ann. Phys. 61(1970), 329.

37. A.J. Macfarlane : Kinematics of the relativistic two-particle system, J. Math. Phys. 4(1963), 490.

38. C. Pirotte : Realizations of the generators of homogeneous Lorentz group, Physica, 63(1973), 373.

39. J.S. Lomont and H.E. Moses : Simple realizations of the infinitesimal generators of the proper orthochronous Lorentz group for mass zero, J. Math. Phys. 3(1962), 405.

40. J. Backers and C. Pirotte : Remarks on poincare generators, Unitary transformations and helicity representations, Nuovo cimento, 64(1969), 439.

41. A. Massiah : Quantum Mechanics, Vol. II Wiley, New York (1962).

42. R. Shaw : Unitary representations of the inhomogeneous Lorentz group. Nuovo Cimento, 33(1964), 1074.

43. H.E. Moses and S.C. Wang : Representatons of the inhomogeneous Lorentz group and the transformation of the Wave function. Nuovo Cimento, 36(1965), 788.

44. A. Chakrabarti : Irreducible unitary representations of Poincare group. J. Math. Phys. 12(1971), 1813.

45. M. Jaspers : On Unitary transformations and generators of lightlike Poincare representations. Nuovo Cimento, 35A (1976), 434.

46. J. Backers and M. Jaspers : On timelike, lightlike and spacelike relizations of Pioncare generators, Ann. Phys, 113(1978), 237.

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Digitized by Arya Samaj Foundation Chennai and eGangotri

47. J.F. Boyce : Irreducible representations of the Poincare group corresponding to spacelike four momentum. J. Math. Phys. 8(1967), 675.

48. L. Sertorio and M. Toller : Three-dimensional Lorentz group and partial wave analysis of the scattering amplitude, Nuovo Cimento, 33(1964), 413.

49. F.T. Hadjoannou : An expansion of the scattering amplitude using the representations of the Lorentz grou, Nuovo Cimento, 44(1966), 153.

50. B.S. Rajput and K.D. Purohit : Reduction of Wave function associated with electro-magnetic field for imaginary mass system to standard helicity representation of Lorentz group, Pramana, 15(1980), 153.

51. B.S. Rajput, K.D. Purohit and O.P.S. Negi : Reduction of wave function which transforms as field associated with spin-zero tachyon, Ind. J. Pure and Appl. Phys. 19(1981), 1081.

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका, खण्ड 12 (1998) अंक 2, 71-82
Journal of Natural & Physical Sciences, Vol. 12 (1998) No.2, 71-82

ON NONLINEAR FUNCTIONAL VOLTERRA INTEGRO DIFFERENTIAL EQUATION

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(Received 30.9.98 revised 30.12.98)

ABSTRACT

The aim of the present paper is to study the global existence result for the nonlinear functional Volterra integro-differential equation of the more general type. An application of the Leray-Schauder alternative is used to establish our result.

INTRODUCTION

Let R^n be the n-dimensional Euclidean space and $|\cdot|$ denotes the Euclidean norm on R^n. Let $C = C([-r,0],R^n), r>0$ denote the Banace space of all continuous functions $\psi:[-r,0]\to R^n$, endowed with the sup-norm

$$\|\psi\| = \sup\{|\psi(\theta)|:\theta\in[-r,0]\}.$$

If x is a continuous function from $[-r,T], T>0$, to R^n and $t\in[0,T]$, then x denotes the element of C given by $x_t(\theta)=x(t+\theta)$ for $\theta\in[-r,0]$. We consider the nonlinear functional Volterra integro-differential equation of the form

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$$\left[x'(t)-h(t,x(t),x_t)\right]' = f\left(t,x(t),x_t,\int_0^t a(t,s)\,g(s,x(s),x_s)ds\right), t\in[0,T], \quad (1.1)$$

$$x(t)=\phi(t), -r\le t\le 0 \text{ and } x'(0)=\alpha, \quad (1.2)$$

where a is measurable real valued function for $t\ge s\ge 0$, α is a real constant, $h,g:[0,T]\times R^n\times C\to R^n$, $f:[0,T]\times R^n\times C\times R$ are continuous functions and $\phi\in C$ is given.

Many recent papers and monographs have dealt with the existence, uniqueness and other properties of solutions of special forms of the equations (1.1)-(1.2), for example, see [2,6,7,8,10] and the references cited therein. In the present paper we study the global existence of solution of the equations (1.1)-(1.2) by using topological transversality theorem of Granas [5], known as Leray-Schauder alternative. The interesting and useful aspect of the method employed here is that it yields simultaneously the existence of a solution and the maximal interval of existence. Recently, in [8], [10] the authors used this method to investigate the global existence of solutions of certain special versions of the above equations, see also [4,9].

STATEMENT OF REUSLT

We use the following version of the topological transversality theorem given by A. Granas in ([5], p.61).

Theorem G : Let B be a convex subset of a normed linear space E and assume $x\in B$. Let $F:B\to B$ be a completely continuous operator and let

$$U(F)=\{x\in B: x=\lambda Fx \text{ for some } 0<\lambda<1\}.$$

Then either $U(F)$ is unbounded or F has a fixed point.

We list the following hypotheses used in our discussion.

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(H_1) There exists a continuous function $p:[0,T] \to R_+ = [0,\infty)$ such that

$$|f(t,u,\psi,v)| \le p(t)(|u| + \|\psi\| + |v|),$$

for every $t \in [0,T], u, v \in R^n$ and $\psi \in C$

(H_2) There exists a constant $c_1 \ge 0$ such that

$$|a(t,s)| \le c_1 \text{ for } t \ge s \ge 0.$$

(H_3) There exists a continuous function $q:[0,T] \to R_+$ such that

$$|q(t,u,\psi)| \le q(t)G(|u| + \|\psi\|)$$

for every $t \in [0,T], u \in R^n$ and $\psi \in C$, where $G: R_+ \to (0,\infty)$ is a continuous nondecreasing function.

(H_4). There exists nonnegative constnats c_2, c_3 and c_4 such that

$$|h(t,u,\psi)| \le c_2 |u| + c_3 \|\psi\| + c_4$$

for every $t \in [0,T]$, $u \in R^n$ and $\psi \in C$.

Our main result is established in the following theorem.

Theorem : Suppose that the hypotheses $(H_1)-(H_4)$ hold. Further suppose that T satisfies the following condition

$$\int_0^T M(s)ds < \int_{3c}^{\infty} \frac{ds}{3s + G\left(\frac{2}{3}s\right)} \tag{2.1}$$

where

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$$c = \left[1 + (c_2 + c_3)T\right]\|\phi\| + (|\alpha| + 2c_4)T \tag{2.2}$$

and $$M(t) = \text{Max}\,\{1, c_2 + c_3, p(t), c_1 q(t)\},\ t \in [0, T]. \tag{2.3}$$

Then the initial value problem (1.1)-(1.2) has a solution x defined on $[-r, T]$.

Proof of the Theorem

First, we establish the priori bounds for the initial value problem $(1.1)_\lambda - (1.2), \lambda \in (0,1)$, where

$$(1.1)_\lambda \left[x'(t) - \lambda h(t, x(t), x_t)\right]'$$

$$= \lambda f\,(t, x(t), x_t, \int_0^t a(t,s) g(s, x(s), x_s) ds), t \in [0, T].$$

Let $x(t)$ be a solution of the initial value problem $(1.1)_\lambda - (1.2)$. Then it satisfies the equivalent integral equation

$$x(t) = \phi\,(0) + [\alpha - \lambda\, h(0, \phi(0), \phi)]\, t + \lambda \int_0^t h(s, x(s), x_s)\, ds \tag{3.1}$$

$$+ \lambda \int_0^t \int_0^s f\left(\tau, x(\tau), x_\tau \int_0^\tau a(\tau, \eta) g(\eta, x(\eta), x_n) dn\right) d\tau ds.$$

Using (3.1) and hypotheses $(H_1) - (H_4)$ we obtain

$$|x(t)| \le c + \int_0^t \left[c_2 |x(s)| + c_3 \|x_s\|\right] ds + \int_0^t \int_0^s p(\tau)\left[|x(\tau)| + \|x_\tau\|\right] d\tau\, ds$$

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$$+\int_0^t \int_0^s p(\tau) \int_0^\tau c_1 q(\eta) G(|x(\eta)| + \|x_\eta\|) d\eta \, d\tau \, ds \tag{3.2}$$

where c is given by (2.2).Let the fnction $z:[-r,T] \to R$ be defined by

$$z(t) = \sup\{|x(s)| : -r \le s \le t\}, -r \le t \le T,$$

and let $t^* \in [-r,t]$ be such that $z(t) = |x(t^*)|$. If $t^* \in [0,t]$ then from (3.2) we have

$$z(t) = |x(t^*)| \tag{3.3}$$

$$\le c + \int_0^{t^*} [c_2|x(s)| + c_3\|x_s\|] \, ds$$

$$+\int_0^{t^*} \int_0^s p(\tau)[|x(\tau)| + \|x_\tau\|] d\tau \, ds$$

$$+\int_0^{t^*} \int_0^s p(\tau) \int_0^\tau c_1 q(\eta) G(|x(\eta)| + \|x_\eta\|) d\eta d\tau \, ds$$

$$\le c + \int_0^t [c_2|x(s)| + c_3\|x_s\|] ds$$

$$+\int_0^t \int_0^s p(\tau)[|x(\tau)| + \|x_\tau\|] d\tau \, ds$$

$$+\int_0^t \int_0^s p(\tau) \int_0^\tau c_1 q(\eta) G(|x(\eta)| + \|x_\eta\|) d\eta d\tau \, ds$$

$$\le c + \int_0^t M(s) z(s) ds + \int_0^t \int_0^s M(\tau) 2z(\tau) d\tau \, ds$$

$$+\int_0^t \int_0^s M(\tau) \int_0^\tau M(\eta) G(2z(\eta)) d\eta \, d\tau \, ds.$$

where $M(t)$ is defined by (2.3). If $t^* \in [-r,0]$ then by using (2.2),

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$z(t) = |x(t^*)| \le \|\phi\| \le c$ and (3.3) holds. Denoting by $u(t)$ the right hand side of (3.3), we have

$$z(t) \le u(t), t \in [0,T], u(0) = c$$

and

$$u'(t) \le M(t)\left[u(t) + \int_0^t M(s)2u(s)ds + \int_0^t M(s)\int_0^s M(\tau)G(2u(\tau))d\tau\, ds\right]$$

Let

$$v(t) = u(t) + \int_0^t M(s)2u(s)ds + \int_0^t M(s)\int_0^s M(\tau)G(2u(\tau))d\tau\, ds$$

then we have

$$u(t) \le v(t),\ t \in [0,T], v(0) = u(0) = c$$

and

$$v'(t) \le M(t)\left[3v(t) + \int_0^t M(s)G(2v(s))ds\right]$$

Define the expression in the square bracket on the right hand side by $w(t)$. Then we have

$$v(t) \le \frac{1}{3}w(t),\ t \in [0,T], w(0) = 3v(0) = 3c$$

and

$$w'(t) \le M(t)\left[3w(t) + G\left(\frac{2}{3}w(t)\right)\right]$$

i.e.

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$$\frac{w'(t)}{3w(t)+G\left(\frac{2}{3}w(t)\right)} \le M(t) \tag{3.4}$$

Integrating (3.4) from 0 to t and by making use of the change of variable and condition (2.1), we obtain

$$\int_{3c}^{w(t)} \frac{ds}{3s+G\left(\frac{2}{3}s\right)} \le \int_0^t M(s)ds \le \int_0^T M(s)ds < \int_{3c}^{\infty} \frac{ds}{3s+G\left(\frac{2}{3}s\right)}. \tag{3.5}$$

By the mean value theorem, there exists a constant β independent of $\lambda \in (0,1)$ such that $w(t) \le \beta$ for $t \in [0,T]$ and hence $v(t) \le \beta$, $u(t) \le \beta$, $z(t) \le \beta$ for $t \in [0,T]$. Since for every $t \in [0,T]$, $\|x_t\| \le z(t)$, we have $\|x\| \le \beta$.

We rewrite the initial value problem (1.1)-(1.2) as follows. For $\phi \in C$, define $\phi \in B, B = C([-r.T], R^n)$ by

$$\overline{\phi}(t) = \begin{cases} \phi(t)\ , -r \le t \le 0, \\ \phi(0)\ , \quad 0 \le t \le T. \end{cases}$$

If $y \in B$ and $x(t) = y(t) + \overline{\phi}(t), t \in [-r,T]$ then it is easy to observe that y satisfies

$$y(t) = y_0 = 0, \ -r \le t \le 0,$$

$$y(t) = [\alpha - h(0,\phi(0),\phi)]t + \int_0^t h\left(s, y(s)+\overline{\phi}(s), y_s + \overline{\phi}_s\right)ds$$

$$+\int_0^t \int_0^s f(\tau, y(\tau)+\overline{\phi}(\tau), y_\tau + \overline{\phi}_\tau,$$

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$$\int_0^{\tau} a(\tau,\eta)g(\eta,y(\eta)+\overline{\phi}(\eta),y_\eta+\overline{\phi}_\eta)d\tau ds.,$$

if and only if x satisfies the equivalent integral equation to (1.1)-(1.2) i.e.

$$x(t)=\phi(0)+[\alpha-h(0,\phi(0),\phi)]t+\int_0^t h(s,x(s),x_s)ds$$

$$+\int_0^t\int_0^s f\left(\tau,x(\tau),x_\tau,\int_0^\tau a(\tau,\eta)g(\eta,x(\eta),x_\eta)d\eta\right)d\tau ds,$$

$$t\in[0,t],x(t)=\phi(t),-r\le t\le 0$$

Define an operator $F:B_0\to B_0, B_0=\{y\in B, y_0=0\}$ by

$$(3.6)\ F\ y(t)=\begin{cases}0, & -r\le t\le 0,\\ [\alpha-h(0,\phi(0),\phi)]t+\int_0^t h(s,y(s)+\overline{\phi}(s),y_s+\overline{\phi}_s)ds\\ +\int_0^t\int_0^s f(\tau,y(\tau)+\overline{\phi}(\tau),y_\tau+\overline{\phi}_\tau,\\ \int_0^\tau a(\tau,\eta)g(\eta,y(\eta)+\overline{\phi}(\eta),y_\eta+\overline{\phi}_\eta)d\eta)d\tau ds, t\in[0,T].\end{cases}$$

Then, clearly F is continuous. Next, we prove that F is completely continuous.

Let $\{w_m\}$ be a bounded sequence in B_0 i.e. $\|w_m\|\ge b$ for all m, where b is a positive constant. Obviously we have $\|w_{m_t}\|\le b, t\in[0,T]$, for all m. Using (3.6) and hypotheses $(H_1)-(H_4)$ and letting $\mu=\sup\{M(t),t\in[0,T]\}$ we have

$$|Fw_m(t)|\le(c_2+c_3)T\|\phi\|+(|\alpha|+2c_4)T$$

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$$+\int_0^t \left[c_2\left|w_m(s)+\overline{\phi}(s)\right|+c_3\left\|w_{m_s}+\overline{\phi}_s\right\|\right]ds$$

$$+\int_0^t\int_0^s p(\tau)\left[\left|w_m(\tau)+\overline{\phi}(\tau)\right|+\left\|w_{m_\tau}+\overline{\phi}_\tau\right\|\right.$$

$$\left.+\int_0^\tau c_1\, q(\eta)G\left(\left|w_m(\eta)+\overline{\phi}(\eta)\right|+\left\|w_{m_\eta}+\overline{\phi}_\eta\right\|\right)d\eta\right]d\tau ds$$

$$\le (c_2+c_3)T\|\phi\|+(|\alpha|+2c_4)T$$

$$+\int_0^t (c_2+c_3)(b+c)ds+\int_0^t\int_0^s p(\tau)[2(b+c)]d\tau ds$$

$$+\int_0^t\int_0^s\int_0^\tau M(\tau)M(\eta)G(2(b+c))d\eta d\tau ds$$

$$\le \mu(b+2c)T+(|\alpha|+2c_4)T+\mu(b+c)T^2+\mu^2G(2(b+c))\frac{T^3}{6}.$$

Hence

$$\|Fw_m\|\le \mu(b+2c)T+(|\alpha|+2c_4)T+\mu(b+c)T^2+\mu^2G(2(b+c))\frac{T^3}{6}.$$

This implies that $\{Fw_m\}$ is uniformly bounded.

Now, we show that the sequence $\{Fw_m\}$ is equicontinuous. To prove this we consider the following cases.

Case (1). $0\le t_1\le t_2$. Then from (3.6), we obtain

$$|Fw_m(t_2)-Fw_m(t_1)|$$

$$\le \int_{t_1}^{t_2}\left[\left[\alpha-h(0,\phi(0),\phi)\right]\right]ds$$

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$$+\int_{t_1}^{t_2}\left|h\left(s, w_m(s)+\overline{\phi}(s), w_{m_s}+\overline{\phi}_s\right)\right| ds$$

$$+\int_{t_1}^{t_2}\int_0^s\left|f\left(\tau, w_m(\tau)+\overline{\phi}(\tau), w_{m_\tau}+\overline{\phi}_\tau,\right.\right.$$

$$\left.\int_0^\tau a(\tau,\eta) g\left(\eta, w_m(\eta)+\overline{\phi}(\eta), w_{m_\eta}+\overline{\phi}_\eta\right) d\eta\right| d\tau ds$$

$$\leq \int_{t_1}^{t_2}\left[|\alpha|+\mu c+c_4\right] ds+\int_{t_1}^{t_2}\left[\mu(b+c)+c_4\right] ds$$

$$+\int_{t_1}^{t_2}\int_0^s \mu\, 2(b+c)\, d\tau ds+\int_{t_1}^{t_2}\int_0^s\int_0^\tau \mu^2 G\left(2(b+c)\right) d\eta d\tau ds.$$

Case (ii) $t_1 \leq 0 \leq t_2$. By using (3.6), we get the estimation as in the case (i) with integrals over $[0, t_2]$ instead of the integrals over $[t_1, t_2]$.

Case (iii) $t_1 \leq t_2 \leq 0$. By using (3.6), we get

$$\left|Fw_m(t_2)-Fw_m(t_1)\right|=0.$$

Since the cases (i), (ii), (iii) imply

$$\left\|Fw_m(t_2)-Fw_m(t_1)\right\| \leq \gamma\left|t_2-t_1\right|$$

with constant $\gamma>0$, for every $t_1, t_2 \in[-r, T], \{Fw_m\}$ is equicontinuous and hence by the Arzela-Ascoli theorem, the operator F is completely continuous.

Moreover, the set

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$$U(F) = \{y \in B_0 : y = \lambda Fy, \lambda \in (0,1)\}$$

is bounded, since for every solution y in $U(F)$ the function $x = y + \overline{\phi}$ is a solution of $(1.1)_\lambda - (1.2)$ for which we have proved $\|x\| \le \beta$ and hence $\|y\| \le \beta + c$. Now, by virtue of Theorem G, the operator F has a fixed point in B_0. Therefore, the initial value problem (1.1)-(1.2) has a solution. Thus, the proof of the theorem is completed.

In concluding this paper, we note our study of the equations (1.1)-(1.2) includes the study of the following control system

$$\left[x'(t) - h(t, x(t), x_t)\right]' = f(t, x(t), x_t) + p(t)u(t), t \in [0,T], \tag{3.7}$$

$$x(t) = \phi(t), t \in [-r, 0] \text{ and } x'(0) = \alpha \tag{3.8}$$

where $u(t)$ is a control given by

$$u'(t) = g(t, x(t), x_t), t \in [0,T], \tag{3.9}$$

$$u'(t) = \psi(t), t \in [-r, 0] \tag{3.10}$$

in which $f, g, h : [0,T] \times R^n \times C \to R^n, p: [0,T] \to R, u: [-r,T] \to R^n$ are continuous functions and $\phi, \psi \in C$ are given. The equation (3.9) is equivalent to the following integral equation

$$u(t) = \psi(0) + \int_0^t g(t, x(s), x_s) ds, \; t \in [0,T]. \tag{3.11}$$

By making use of (3.11) in (3.7), we get the following integro-differential equation

$$\left[x'(t) - h(t, x(t), x_t)\right]' \tag{3.12}$$

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$$= f(t, x(t), x_t) + p(t)\left[\psi(0) + \int_0^t g(s, x(s), x_s)ds\right].$$

The equation (3.12), with (3.8) is a special case of the equations (1.1)-(1.2). Thus, the study of equations (1.1)-(1.2) includes the study of control system of the form (3.7) - (3.10).

ACKNOWLEDGEMENT

The author expresses his sincere gratitude to Professor B.G. Pachpstte for many helpful discussions and suggestions during the preparation of the paper.

REFERENCES

1. L.E. Bobisud and D.O'regan, Existence of solutions to some singular initial value problems, J. Math.Anal.Appl. 133(1998) 214-230.
2. C. Corduneanu, Integral Equations and Stability of Feedback Systems, Academic Press, New York, 1973.
3. M.B. Dhakne, On a nonlinear functional integro-differential system, J. Natur. Phys. Sci. Vol. 5(1991-94), 123-137.
4. M.B. Dhakne, Global existence of solutions of nonlinear functional integral equations, accepted for publication in *Indian J. Pure Appl. Math.*
5. J. Dugundji and A. Granas, Fixed Point Theory, Vol. I, Monografie Matematyczne, PWN, Warsaw, 1982.
6. J.K. Hale, Theory of Functional Differential Equations, *Springer-Verlag*, New York, 1977.
7. S.K. Ntouyas, Y.G. Sficas and P. Ch. Tsamatos, Existence results for initial value problems for neutral functional differential equations, *J. Differential Equations* 114(1994), 527-537.
8. S.K. Ntouyas and P. Ch. Tsamatos, Global existence for functional integrodifferential equations of delay and neutral type, Applicable Anal. 54(1994), 251-262.
9. B.G. Pachpatte, Applications of the Leray-Schauder alternative to some Volterra integral and integro-differential equations, Indian J. Pure Appl. Math. 26(12), (1995), 1161-1168.
10. B.G. Pachpatte, Global existence results for certain integro-differential equations, Demonstratio Mathematica Vol. XXIX (1996), 23-30.

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प्राकृतिक एवं भौतिकीय विज्ञान शोध पत्रिका, खण्ड 12 (1998) अंक 2, 83-89
Journal of Natural & Physical Sciences, Vol. 12 (1998) No.2, 83-89

WEIGHTED CONVOLUTION ALGEBRAS ON A LOCALLY COMPACT ABELLIAN GROUP

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(Received 10.5.1998†)

ABSTRACT

In the present paper we prove that the weighted space $L^1_{\omega^r}(G), 0 \le r \le 1, G$ being a metrizable locally compact abelian group is a Banach algebra if and only if the weight function $w(x), x \in G$, is submultiplicative. Our theorem includes the corresponding result of Grabiner [1] as particular case for $r = 1$ and $G = [0, \infty]$.

A.M.S. Classification : 43 A 15, 43 A 20.
Keywords : Weighted, Convolution algebras, Metrizable locally compact abelian group.

INTRODUCTION

Let G be a metrizable locally compact abelian group with the normalized Haar measure dx and translation-invariant metric d such that $|x| = d(0, x), x \in G$.

We suppose that Ω is the set of all functions ω satisfying the conditions :

(i) ω is a measurable function on G and positive almost everywhere for the Haar measure dx.

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† Corrected Proof received after finalising other sequence.

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(ii) ω^r *and* ω^{-r} are locally integrable functions for $0 \le r \le 1$.

(iii) $\omega^r(x+y) \le \omega^r(x)\omega^r(y)$ for all $x, y \in G$.

We suppose that $L^p_{\omega^r}(G)$ denotes the space of all equivalence classes of complex-valued measurable functions f *on* G such that

$$\|f\|_{p,\omega^r} = \left[\int_G |f(x)|^p \omega^r(x)dx\right]^{1/p} < \infty . \tag{1.1}$$

The space $L^1_{\omega^r}(G)$ is a Banach space with respect to the norm (1.1). The conjugate of $L^{-p}_{\omega^r}(G)$ is the space $L^{p'}_{\omega'}(G)$, where $1/p + 1/p' = 1$ and $\omega' = \omega^{r/(1-p)}$.

It can be easily seen that $L^p_{\omega^r}(G)$ is a reflexive Banach space for $1 < p < \infty$.

The dual space $L^1_{\omega^r}(G)$ is the space $L^\infty_{\omega^r}(G)$, which is the Banach space of complex-valued measurable functions f on G under the norm.

$$\|f\|_\infty = ess \sup \{|f(x)|/w^r(x) : x \in G, 0 \le r \le 1\}$$

For each ω^r, let $C_{w^r}(G)$ be the class of all function g such that $g\omega^r \in C_0(G)$, where $C_0(G)$ is the space of all continuous function on G which vanish at infinity.

Following Grabiner [1,p.553], we say that $x \in G$ is a Lebesgue point of f provided

$$\lim_{h\to 0} 1/h \int_u osc_u f(x)dx = 0,$$

where $U = \upsilon_x$ is a compact neighbourhood of u such that $|U| = h$ and

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$$Osc_U = \sup_{u \in U} |f(x+u) - f(x)|.$$

Now our aim is to prove the following :

Theorem : If $\omega \in \Omega, 0 \le r \le p$ and $1 \le p \le \infty$, then the following are equivalent.

i) $L^1_{\omega^r}(G)$ is an algebra.

ii) $L^1_{\omega^r}(G) * L^p_{\omega^r}(G) \le L^p_{\omega^r}(G)$.

iii) w is weakly submultiplicative.

In case G is the half line $(0,\infty)$ and $r = p$, then our theorem reduced to Theorem 2.1 of Grabiner [1,p.553].

The proof of the theorem depends on the following :

Lemma 2.1: For $\omega \in \Omega$, the following conditions are equivalent :

i) $\|f * g\|_{p,\omega^r} \le C \|f\|_{1,\omega^r} \|g\|_{p,\omega^r}$, *where* $f \in L^1_{\omega^r}(G)$

and $g \in L^p_{\omega^r}(G)$.

ii) $w(x+y) \le C\, w(x)\, w(y)$, where x, y and $x+y$ are Lebesgue points of w and C is a positive constant, not necessarily the same at each occurence.

Proof : At first we shall show that

(i) $\rightarrow$ (ii)

On the lines of Grabiner [1, p33], we suppose that u is an element of G such that

$$\lim_{h \to 0} 1/h \int_u Osc_U f(x)\, dx = 0, \tag{2.1}$$

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where $U \equiv U_x$ is a compact neighbourhood of u such that $|U| = h$ and

$$Osc\upsilon = \sup_{u \in U} |f(x+u) - f(x)|.$$

Thus u is a Lebesgue point of f.

Let a, b and $a+b$ be Lebesgue point of ω. We assume that $f_h(x)$ and $g_h(x)$ are characteristic functions of U_a and U_b respectively such that

$$f_h * g_h = \delta_{a+b} * K_h, \tag{2.2}$$

where δ_{a+b} is the unit measure at $a+b$ and

$$\delta_{a+b} = \begin{cases} x/2 \ in \ U_a \cup U_b \\ 0 \ \text{elsewhere.} \end{cases} \tag{2.3}$$

Thus we have

$$\delta_{a+b} * K_h/h^2 = f_h/h * g_h/h,$$

which implies that

$$\begin{aligned} \|\delta_{a+b} * K_h/h^2\|_{1,\omega^r} &= C\|f_h/h\|_{1,\omega^r}\|g_h/h\|_{1,\omega^r} \\ &\to C\omega(a)\,\omega(b) \\ &as\, h \to 0. \end{aligned} \tag{2.4}$$

Again using the method of Grabiner, we have

$$\begin{aligned} \|\delta_{a+b} * K_h/h^2\|_{1,\omega^r} - \omega(a+b) \\ &= 1/h^2 \int_{U_2 \cup U_b} K_h(x)[\omega(x+a+b) - \omega(a+b)]dx \\ &= 1/h^2 \int_{U_a \cup U_b} x/2\,[\omega(x+a+b) - \omega(a+b)]dx \end{aligned}$$

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$$= 1/h^2 . 2h/2 \int_{U_a \cup U_b} x/2\, [\omega(x+a+b) - \omega(a+b)]\, dx$$

$$\to \theta(a) \qquad (2.5)$$

$$as\, h \to 0$$

Combining (2.4) and (2.5), we infer that

$$\omega(a+b) \leq \omega(a)\,\omega(b)$$

Hence the lemma holds true. We now prove that ii) $\to$ i)

Let us assume that $\omega(a+b) \leq \omega(a)\,\omega(b)$,

where a,b and (a+b) are Lebesgue points. But almost every x in G is a Lebesgue point of ω and for a fixed x almost every $y \in G$ is a Lebesgue point of ω. Hence $x+y$ is a Lebesgue point of w. Thus, by Fubini's theorem, the inequaility

$$\omega(x+y) \leq \omega(x)\,\omega(y)$$

holds for almost every x, y.

Hence as in [1], we have

$$|f * g|(x)\, \omega^r(x) \leq C |f\, \omega^r| * |g\, \omega^r|$$

which implies that

$$\|f * g\|_{p,\omega^r} \leq C \|f\|_{1,\omega^r} \|g\|_{p,\omega^r}$$

This completes the proof of the lemna.

PROOF : At first we show that i) $\to$ ii)

Since $L^1_{\omega^r}(G)$ is an algebra and

$L^p_{\omega^r}(G) \subseteq L^1_{\omega^r}(G)$, therefore $L^p_{\omega^r}(G)$ is a closed ideal under convolution in $L^1_{\omega^r}(G)$ and

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$$\|f*g\|_{p,\omega^r} \le C\|f\|_{1,\omega^r}\|g\|_{p,\omega^r}$$

where

$$f \in L^1_{\omega^r}(G) \text{ and } g \in aL^p_{\omega^r}(G)$$

Thus we infer that

$$L^1_{\omega^r}(G) * L^p_{\omega^r}(G) \subseteq L^p_{\omega^r}(G)$$

Next, we prove that ii) $\rightarrow$ iii)

Under the hypothesis ii), we have

$$\|f*g\|_{p,\omega^r} \le C\|f\|_{1,\omega^r}\|g\|_{p,\omega^r}, \text{ where } f \in L^1_{\omega^r}(G) \text{ and } g \in aL^p_{\omega^r}(G).$$

Let a, b and $a+b$ be Lebesgue points of ω. Sionce $\omega^r, \theta \le r \le p, 1 \le p \le \infty$, is positive and locally bounded; a, b and $a+b$ are Lebesgue points of ω^r. Now, as in Lemma 2.1, we have

$$\left\|\delta_{a+b} * K_h/(h^{1+1/p})\right\|_{p,\omega^r} = C\|f_h/h\|_{1,\omega^r}\|g_h/h\|_{1,\omega^r}$$

$$\longrightarrow \omega^r(a)\,\omega^r(b) \text{ as } h \longrightarrow 0 \tag{3.1}$$

Next, since

$$1/h^{p+1}\int_{U_a \cup U_b} K_h^p(x) = 2/p+1,$$

we have

$$\lim_{h\to\infty}\left\|\delta_{a+b} * K_h/(h^{1+1/p})\right\|_{p,\omega^r} = [2/p+1]^{1/p}\,\omega(a+b) \tag{3.2}$$

Combining (3.1) and (3.2), we get

$$\omega^r(a+b) \le C[2/p+1]^{1/p}\,\omega^r(a)\,\omega^r(b),$$

which implies that

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$$\omega(a+b) \leq C\,\omega(a)\,\omega(b).$$

Hence ω is submultiplicative.

Finally, it remains to show that iii) $\Rightarrow$ i).

Following Grabiner [1,p. 534] and using te hypothesis iii), we have

$$|f*g|(x)\,\omega^{r/p}(x) \leq C\left|f\omega^{r/p}\right|*\left|g\omega^{r/p}\right|,$$

which implies that

$$\|f*g\|_{1,\omega^r} \leq C\,\|f\|_{1,\omega^r}\|g\|_{1,\omega^r}$$

Hence we see that $L^1_{\omega^r}(G)$ is closed with respect to convolution and, therefore, it forms a Banach algebra.

This complets the proof of the Theorem.

REFERENCE

1. Sandy Grabinber : Weighted convolution algebra on the half line, J. Math. Annl. Appl. 83(1981), 531-533.

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फार्म-4

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मैं एस. एन. सिंह, कुलसचिव, गुरुकुल कांगड़ी विश्वविद्यालय, हरिद्वार घोषित करता हूँ कि उपरिलिखित तथ्य मेरी जानकारी के अनुसार सही हैं।

हस्ताक्षर

एस. एन. सिंह
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